

Designing frequency selective filters and analyzing their effects on frequency modulated signals

For this lab, you will design some frequency selective filters. Instead of using generalized frequency, ranging from 0 to $\frac{1}{2}$, though used, we will use “physical” frequency, ranging from 0 to half the sampling frequency (in Hz).

We’ll also use a new broadband signal: a frequency modulation (FM) sweep, to see the results of your filters. In contrast to amplitude modulation (AM), which changes the amplitude of the carrier (original signal), the *frequency* of the carrier signal is changed. An FM sweep has a single instantaneous frequency that changes in time. For us, this will mean also that the effects of frequency selective filters will occur not only at specific frequencies but also at specific times.

Finally, we’ll also display the results using spectrograms. We haven’t covered spectrograms in class yet, but basically they are very useful when the spectrum (frequency composition) of a signal changes in time.

For this lab, use a sampling frequency of $f_s = 1000$ Hz, a total time of $T = 2$ seconds, $N = f_s * T$ total points, $n = [0:N-1].'$ samples, and $t = n/f_s$ time steps. (What is the fastest signal that can be represented with this f_s ?)

- 1) For the FM sweep, use $f_m = 1$ Hz and create the signal $x = \cos(2 * \pi * (f_m / f_s) * n.^2 / 2)$; . It starts off with low frequency oscillations, linearly increases in frequency until the oscillation frequency is half the sampling frequency, and then it goes down to zero, back up to $f_s/2$, and then back to zero.
 - a) Examine the signal as a function of time using `plot(t, x)`. Does the signal have the behavior indicated above (roughly)? It may be hard to see.
 - b) Examine the time-varying spectrum, with `spectrogram(x, 64, 32, 64, fs, 'yaxis');` `colorbar`. Does it agree with the behavior indicated above? At what times (roughly) is the frequency content dominated by 100 Hz? 200 Hz? 300 Hz? 400 Hz? 500 Hz?
 - c) Listen to the signal using `soundsc(x, fs)`. Does it agree with the behavior indicated above?

- 2) Create a lowpass elliptical filter with the following properties:
`fpass=100; fstop=300; rpass = .5; rstop = 60,`
where the first two are in physical frequency units (Hz) not generalized frequency.
 - a) What order is the filter?
 - b) Plot the filter using `freqz(b, a, N, fs)`. Does it have the desired boundary frequencies?
 - c) Process the input signal x into the output signal y by using `filter()`. Given the time-changing

frequency content of the filter, especially your answers to (1b), at which times should the sound of the output be attenuated?

- d) Examine the filtered signal as a function of time using `plot(t,y)`. When is the signal attenuated?
 - e) Examine the time-varying spectrum, with `spectrogram(y,64,32,64,fs,'yaxis');` `colorbar`. For which frequencies is the signal attenuated? Which times? Does this agree with above?
 - f) Listen to the filtered signal using `soundsc(y,fs)`. Can you hear the signal attenuated at the (roughly) correct frequencies? At the (roughly) correct times?
- 3) Repeat (2) but for a bandpass elliptical filter with `fpass=[200 300]; fstop= [100 400]`.
- a) b) c) d) e) f)
- 4) Repeat (2) but for a highpass elliptical filter with `fpass=400; fstop= 300`.
- a) b) c) d) e) f)
- 5) Repeat (2) but for a bandstop elliptical filter with `fpass=[100 400]; fstop= [200 300]`.
- a) b) c) d) e) f)
- 6) *Bonus* Create a notch filter with notch frequency $f_n = 200$ Hz and notch-width parameter $a_0 = 0.975$ (the closer it is to 1, the narrower the notch, but the greater the side-effects), using this Matlab code:

```
b1 = [1; -exp(j*2*pi*fn/fs)]
a1 = [1; -a0*exp(j*2*pi*fn/fs)]
b = real(conv(b1,conj(b1)))
a = real(conv(a1,conj(a1)))
```

- a) Plot the filter as in (2b). Does it have the desired characteristic frequency?
- b) Process the input signal x into the output signal y by using `filter()`. Given the time-changing frequency content of the signal, especially your answers to (1b), at which times should the sound of the output be attenuated?
- c) Examine the filtered signal as a function of time using `plot(t,y)`. When is the signal attenuated?
- d) Examine the spectral content as a function of time as in (2e). For which frequencies is the signal attenuated? Which times? Does this agree?
- e) Listen to the filtered signal as in (2f). Can you hear the signal attenuated at the (roughly) correct frequency? At the (roughly) correct times?
- f) Can you see the effect of the group delay in the spectrogram? Can you remove it by using `filtfilt()` instead of `filter()`? Why not?