BIOL 708L/NACS 728B Homework #5
Fall 2011

1) Re-express the following complex numbers, without using Matlab. If asked to express the number as its real and imaginary components, your answer should be of the form: $z = a + jb$ or $\mathrm{Re}(z) = a,\ \mathrm{Im}(z) = b$, for your particular answers of a and b (either one of which might be zero). If asked to express the number as its magnitude and phase, your answer should be of the form: $z = A\mathrm{e}^{j\theta}$ or $|z| = A,\ \angle z = \theta$ for your particular answers of A and θ. You may express phase in radians or degrees, whichever you feel more comfortable with, but if you choose radians, please answer in fractions of π (e.g. π /2, not 1.57)

a) The real and imaginary components of 1.
b) The real and imaginary components of *j*.
c) The real and imaginary components of -*j*.
d) The real and imaginary components of e^{j0}
e) The real and imaginary components of $\mathrm{e}^{j\pi}$
f) The real and imaginary components of $\mathrm{e}^{j\frac{1}{2}\pi}$
g) The real and imaginary components of $\mathrm{e}^{-j\frac{1}{2}\pi}$
h) The real and imaginary components of $\mathrm{e}^{-j\pi}$
i) The real and imaginary components of $\mathrm{e}^{3j\pi}$
j) The real and imaginary components of $4\mathrm{e}^{j\frac{1}{2}\pi}$
k) The real and imaginary components of $\sqrt{2}\,\mathrm{e}^{j\frac{1}{4}\pi}$ (*You might need a calculator for this one*.)
l) The real and imaginary components of $\sqrt{2}\,\mathrm{e}^{j\frac{3}{4}\pi}$ (*You might need a calculator for this one*.)
m) The magnitude and phase of 1.
n) The magnitude and phase of *j*.
o) The magnitude and phase of -*j*.
p) The magnitude and phase of -*j*.
q) The magnitude and phase of 1+*j*.
r) The magnitude and phase of -1+*j*.
s) The magnitude and phase of 2+*j*. (*You might need a calculator for this one*.)
t) The magnitude and phase of 2-*j*. (*You might need a calculator for this one*.)
u) The magnitude and phase of $\mathrm{e}^{j\frac{1}{2}\pi}$.
v) The magnitude and phase of $2\mathrm{e}^{j\frac{1}{6}\pi}$.

2) Like Lab #5, the main goal of this question is to make signals in Matlab and see what their Fourier Transform looks like. Make sure you understand what's going on with the signals at each step. For the stated N, define n to be a ***column*** vector that starts from zero and runs for N points (though for all graphs in the time domain, n will still run along the horizontal axis). Make another column array of points corresponding to the Fourier frequency. Call it f, and it should takes values from 0 to (just less than) 1, should be N points long, starting at 0, in steps of $1/N$. In all plots in the frequency domain, make sure you put f (not "k") along the horizontal axis. Please hand in your plots and answers to your questions (Matlab code is optional).

Let the frequency f_A be 1/8 and f_B be 1/4 (= 2/8). Let N be 32.

a) Define signal 1: `sig1 = cos(2*pi*fA*n-pi/4)`, a sinusoid which is neither a sine nor a cosine. Open up a figure and graph the following (using `stem()` or `bar()`) in 5 subplots: `sig1` itself, the real part of its Fourier transform, the imaginary part of its Fourier transform, the magnitude of its Fourier transform, the phase of its Fourier transform.

b) Of the real part, imaginary part and magnitude of the Fourier transform, which components are non-zero? What frequencies do they correspond to? What are their values? How are they related to f_A?

c) For the phase of the Fourier transform we need only examine the components whose magnitude is non-zero. Why? What is the phase of those components? Knowing the phase and the magnitude of those components, calculate the full value of the Fourier transform at those points. Does it agree with what the actual value of the Fourier transform should at those points? With the real and imaginary parts of the Fourier transform at those points?

d) Repeat (a)-(c) for signal 2: `sig2 = cos(2*pi*fA*n) + sin(2*pi*fB*n)`. You will now need to consider how the frequencies of the non-zero magnitude components compare to f_B as well as to f_A.