Chapter 4, Part 2
Bonds, Bond Prices, Interest Rates and Holding Period Return

Bond Prices

1. Zero-coupon or discount bond
   - Promise a single payment on a future date
   - Example: Treasury bill

2. Coupon bond
   - Periodic interest payments + principal repayment at maturity
   - Example: U.S. Treasury Bonds and most corporate bonds

3. Consol
   - Periodic interest payments forever, principal never repaid
   - Example: U.K. government has some outstanding

Zero-Coupon Bonds

- Straightforward type of bond. U.S. Treasury bills (T-bills) are a good example - money market instruments that mature is less than a year.
  - Each T-bill represents a promise by the U.S. government to pay $1000 at maturity on a fixed future date.
  - No coupon payments - zero-coupon bonds
  - Also called discount bonds since the bond price is less than face value - they sell at a discount.

\[ P = \frac{SFV}{(1+i)^n} \]
Zero-Coupon Bonds

How to price a $1000 face value zero-coupon bond -

\[ P = \frac{1000}{(1+i)^n} \]

Assume \( i = 5\% \), this is an annual yield:

- Price of a One-Year Treasury Bill
  
  \[ \frac{1000}{(1+0.05)} = 952.38 \]

- Price of a Six-Month Treasury Bill
  
  \[ \frac{1000}{(1+0.05)^{1/2}} = 975.90 \]

Remember units must match: \( n \) must be annual since \( i \) is annual – in this case \( n \) equals \( 1/2 \) of a year.

Zero-Coupon Bonds

- Given the Price and the Face Value, we can compute the interest rate using the present value formula.
- Suppose a 1-year T-Bill has a face value of $1000 and the price is $950.
  
  \[ i = \frac{FV}{P} - 1 \]
  
  \[ i = (\frac{1000}{950}) - 1 = 0.0526 = 5.26\%. \]

Price of a Coupon Bond

\[ P_{CB} = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \cdots + \frac{C}{(1+i)^n} + \frac{FV}{(1+i)^n} \]

\[ P_{CB} = \text{Present Value of yearly coupon payments (C) + Present Value of the Face Value (FV),} \]

where: \( i \) = interest rate

\( n \) = time to maturity

C is contractually fixed.
Example: Price of a 10%, n-year Coupon Bond

Coupon Payment (C) = $100, Face value (FV) = $1,000, and $n$ = time to maturity

\[ P_B = \frac{100}{(1+i)^1} + \frac{100}{(1+i)^2} + \ldots + \frac{100}{(1+i)^n} + \frac{1000}{(1+i)^n} \]

Given values for $i$ and $n$, we can determine the bond price $P_{CB}$

**Definition:** Coupon Rate = Coupon Payment / Face Value

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Example, Price of a 10-year Coupon Bond

If $n = 10$, $i = 0.10$, $C = $100 and FV = $1000.

\[ P_{CB} = $1000 = \left[ \frac{100}{(1+.1)^1} + \frac{100}{(1+.1)^2} + \ldots + \frac{100}{(1+.1)^{10}} \right] + \frac{1000}{(1+.1)^{10}} \]

What’s the Coupon Rate in this example?

**Excel Example**

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Price of a 10-year Coupon Bond

If $n = 10$, $i = 0.12$, $C = $100 and FV = $1000.

\[ P_{CB} = $887 = \left[ \frac{100}{(1+.12)^1} + \frac{100}{(1+.12)^2} + \ldots + \frac{100}{(1+.12)^{10}} \right] + \frac{1000}{(1+.12)^{10}} \]

$i$ goes up and $P_{CB}$ goes down

What’s the coupon rate for this bond?
Yield to Maturity - YTM

• **yield to maturity:**
  - The yield bondholders receive if they hold the bond to its **maturity** when the final principal payment is made.
  - Simple example:
    Suppose a $100 face value, 1-yr, 5% coupon bond sells for $100:
    \[ P = \frac{\$5}{(1+i)} + \frac{\$100}{(1+i)} = \frac{\$105}{(1+i)} \]
    • The value of \( i \) that solves the equation is the yield to maturity (YTM).

Example:
1 year, $100 FV, 5% coupon bond selling for $99

**Yield to maturity** for this bond is 6.06 percent found as the solution to:
\[ \frac{\$99}{(1+i)} = \frac{\$105}{(1+i)} \]
\[ (1+i) = \frac{(\$105)}{(\$99)} = 1.0606 \Rightarrow i = 1.0606 - 1 = .0606 \]

Yield to Maturity - YTM

• If you pay $99 for a $100 face value bond, you will receive both the interest payment and the increase in value from $99 to $100.

• This rise in value is referred to as a **capital gain** and is part of the return on your investment.

• When the price of a bond is higher than face value, the bondholder incurs a **capital loss**.
Current Yield

Current yield is C/P:

1 year, $100 FV, 5% coupon bond selling for $99:

Current Yield = \( \frac{5}{99} = 0.0505 \), or 5.05%

Recall:

\[ \text{YTM} = \left( \frac{\text{FV} - \text{P}}{\text{P}} \right) \times (1 + i)^n \Rightarrow i = 1.0606 - 1 = 0.0606 \text{, or } 6.06\% \]

YTM: 10-year Coupon Bond

Suppose \( n = 10 \), \( P_B = $950 \), \( C = $100 \) and \( FV = $1000 \).

\[
\text{P}_B = $950 = \left[ \frac{$100}{(1+i)^1} + \frac{$100}{(1+i)^2} + \ldots + \frac{$100}{(1+i)^{10}} \right] + \frac{$1000}{(1+i)^{10}}
\]

What is the Coupon Rate?

What is the Current Yield?

What’s the YTM? - more complicated

- YTM = .1085 or 10.85%

Approx\(\text{YTM} \approx \frac{C + \frac{FV - P}{n}}{\frac{FV + P}{2}}\)

\( C = \) the coupon payment

\( FV = \) Face Value

\( P = \) Price

\( n = \) years to maturity
Using the Approximation Formula

Previous example:

\[ n = 10, P_B = \$950, C = \$100 \text{ and } FV = \$1000. \]

\[ \text{Approx YTM} = \frac{100 + \frac{50}{10}}{\frac{1950}{2}} = \frac{105}{975} = .1077 \]

\[
\text{Current Yield} = \frac{\text{Yearly Coupon Payment}}{\text{Price Paid}}
\]

For our 10-year, $1000 FV bond with a $100 coupon payment selling at $950:

- Coupon rate = 10%
- Current Yield = 10.52%
- YTM = 10.85%
- Approx. YTM = 10.77%

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of Bond ($)</td>
<td>Yield to Maturity (%)</td>
</tr>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.46</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate
- The price of a coupon bond and the yield to maturity are negatively related
- The yield to maturity is greater than the coupon rate when the bond price is below its face value
More on Zero Coupon or Discount Bonds

**Definition:** A **discount bond** is sold at some price $P$, and pays a larger amount ($FV$) after $t$ years. There is no periodic interest payment.

Let $P_B =$ price of the bond, $i =$ interest rate, $n =$ years to maturity, and $FV =$ Face Value (the value at maturity):

$$P_B = \frac{FV}{(1+i)^n}$$

Zero Coupon Bonds - Price

Price of a One-Year Treasury Bill at 4% and $FV =$ $1,000:

$$P_B = \frac{1000}{(1 + 0.04)} = \$961.53$$

Price of a Six-Month Treasury Bill at 4% and $FV =$ $1,000:

$$P_B = \frac{1000}{(1 + 0.04)^{\frac{1}{2}}} = \$980.58$$

Price of a 20-Year zero coupon bond at 8% and $FV =$ $20,000:

$$P_B = \frac{20000}{(1 + 0.08)^{20}} = \$4,290.96$$

YTM - Zero Coupon Bonds

$$P = \frac{FV}{(1+i)^n}$$

$$(1+i)^n = \left(\frac{FV}{P}\right)^{\left(\frac{1}{n}\right)} \Rightarrow 1 + i = \left(\frac{FV}{P}\right)^{\left(\frac{1}{n}\right)} \Rightarrow i = \left(\frac{FV}{P}\right)^{\left(\frac{1}{n}\right)} - 1$$
Zero Coupon Bonds - YTM

• For a discount bond with FV = $15,000 and P = $4,200, and n = 20, the interest rate (or yield to maturity) would be:

\[ i = \left( \frac{FV}{P} \right)^{\frac{1}{n}} - 1 \]

\[ i = \left( \frac{15,000}{4,200} \right)^{\frac{1}{20}} - 1 \]

\[ i = 1.0657 - 1 \Rightarrow i = 6.57\% \]

Note: This is the formula for compound annual rate of growth

Zero Coupon Bonds - YTM

• For a discount bond with FV = $10,000 and P = $6,491, and n = 7, the interest rate (or yield to maturity) would be:

\[ i = \left( \frac{FV}{P} \right)^{\frac{1}{n}} - 1 \]

\[ i = \left( \frac{10,000}{6,491} \right)^{\frac{1}{7}} - 1 \]

\[ i = 1.06368 - 1 = 0.06368 \text{ or } 6.368\% \]

http://online.wsj.com/mdc/public/page/2_3020-tstrips.html

From a Coupon Bond to Zero Coupon Bonds

Zero Coupon Bonds are called Strips – here’s why.

\[ P = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{FV}{(1+i)^n} \]

Create \( n+1 \) discount bonds

http://www.treasurydirect.gov/instit
/marketable/strips/strips.htm
Consols

- **Consols** or **perpetuities**, are like coupon bonds whose coupon payments last forever.
- The borrower pays only interest, never repaying the principal.
- The U.S. government sold consols once in 1900, but the Treasury has bought them all back.
- The price of a consol is the present value of all future coupon payments.

\[
P_{\text{consol}} = \frac{\text{Yearly Coupon Payment}}{i}
\]

\[
i = YTM = \frac{\text{Yearly Coupon Payment}}{P_{\text{consol}}}
\]

Holding Period Return

- The **holding period return** is the return to holding a bond and selling it before maturity.

- The holding period return can differ from the yield to maturity.

One Year Holding Period Return

Example:

- 10 year bond
- 6% coupon rate
- Purchase at face value, $100
- Hold for one year and then sell it

- What additional information do you need to answer this question?
Holding Period Return

• Suppose market interest rates at the time of the sale fall to 5%?

• 1-yr Holding Period Return =

\[
\frac{6}{100} + \frac{107.11 - 100}{100} = \frac{13.11}{100} = 0.1311
\]

Current Yield + Capital Gain

The investor earned $13.11 on a $100 investment.
The 1-yr Holding Period Return = 13.11%

Holding Period Return

What if market interest rates at the time of the sale rise to 7%?

\[
\frac{6}{100} + \frac{93.48 - 100}{100} = -\frac{0.52}{100} = -0.0052
\]

1-yr Holding Period Return = - 0.52%

Holding Period Return

• The one-year holding period return is the sum of the yearly coupon payment divided by the price paid for the bond and the change in the price divided by the price paid.

\[
= \frac{\text{Yearly Coupon Payment}}{\text{Price Paid}} + \frac{\text{Change in Price of the Bond}}{\text{Price of the Bond}}
= \text{Current Yield} + \text{Capital Gain (as a %)}
\]
Holding Period Return

- You purchase coupon bond and sell one year later.

\[
RET_{t-1\to t+1} = \frac{C}{P_t} + \frac{(P_{t+1} - P_t)}{P_t} = \frac{C + P_{t+1} - P_t}{P_t}
\]

Current Yield  
Capital Gain

Another Example

You purchase a 10-year, 10% coupon bond, face value = $1,000. If the interest rate one year later is the same at 10%:

One year holding period return =

\[
\frac{$100}{$1000} + \frac{$1000 - $1000}{$1000} = \frac{$100}{$1000} = .10
\]

or 10.0%

Holding Period Return

If the interest rate one year later is lower, say at 8%:

One year holding period return =

\[
\frac{$100}{$1000} + \frac{$1125 - $1000}{$1000} = \frac{$225}{$1000} = .225
\]

or 22.5%

- Where did the $1125 come from?
- In the previous example, where did the $107.11 come from?
Holding Period Returns

If the interest rate in one year is higher at 12%:

One year holding period return =

\[
\frac{$100}{$1000} + \frac{$893 - $1000}{$1000} = \frac{-$7}{$1000} = -0.007
\]

or -7.0%

Where did the $893 come from. You need to know how to calculate the $1125 and $893

Key Conclusions From Table

- The return equals the yield to maturity (YTM) only if the holding period equals the time to maturity.
- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if the holding period is less than the time to maturity.
- The greater the percentage price change associated with an interest-rate change, the more distant the maturity.
Interest-Rate Risk

- Change in bond price due to change in interest rate
- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds
- There is no interest-rate risk for a bond whose time to maturity matches the holding period

Reinvestment (interest rate) Risk

- If investor’s holding period exceeds the term to maturity
  - proceeds from sale of bond are reinvested at new interest rate
  - the investor is exposed to **reinvestment risk**
- The investor benefits from rising interest rates, and suffers from falling interest rates