

**Math/Cmsc 475, Jeffrey Adams**  
**Generating Permutations**

Here is algorithm L for generating all permutations of a set or multiset, from *The Art of Computer Programming, Volume 4A, page 319* by Donald Knuth.

Write the set in ascending order  $x_1 \leq x_2 \leq \dots \leq x_n$ . Note that some of these can be equal.

- (1) Find the largest  $j$  such that  $x_j < x_{j+1}$ .
- (2) Find the largest  $k$  such that  $x_j < x_k$ .
- (3) Swap  $x_j$  and  $x_k$ .
- (4) Reverse all  $x_{j+1}, \dots, x_n$ .

For example consider 1, 2, 2, 3

- (1)  $j = 3, k = 4$ , swap the (second) 2 and 3: 1, 2, 3, 2. There is no reversing to do.
- (2)  $j = 2, k = 3$ , swap the (first) 2 and the 3 to get 1, 3, 2, 2, then reverse 2, 2 to (still) get 1, 3, 2, 2.
- (3)  $j = 1, k = 4$ , swap 1 and the (second) 2, to get 2, 3, 2, 1, then reverse the last 3: 2, 1, 2, 3.
- (4)  $j = 3, k = 4$ : swap the last two: 2, 1, 3, 2.
- (5)  $j = 2, k = 4$ , swap 1, 2, to get 2, 2, 3, 1, reverse the last two: 2, 2, 1, 3.

Final result:

1223 1232 1322 2123 2132 2213 2231 2312 2321 3122 3212 3221.

A trivial example: 1112. You get: 1121, 1211, 2111.