**Math 341, Jeffrey Adams**
Test I, March 18, 2011
For complete credit you must show all work
Each problem is worth 20 points

(1)
(a) Suppose $f(x,y,z)$ is a twice differentiable function. Show that $\text{curl}(\nabla f) = 0$.
(b) Suppose $\mathbf{F}(x,y,z)$ is a (continuously differentiable) vector field defined in an open set $S$ in $\mathbb{R}^3$, and $\text{curl}(\mathbf{F}) = 0$. Is there necessarily a function $f(x,y,z)$ such that $\nabla f = \mathbf{F}$ in $S$? Justify your answer.

(2) Suppose $\alpha > 0$ is a constant, and

$$\mathbf{F}_\alpha(x,y) = (\frac{-y}{(x^2+y^2)^\alpha}, \frac{x}{(x^2+y^2)^\alpha})$$

(a) Compute $\int_\gamma \mathbf{F}_\alpha \cdot \mathbf{dx}$ where $\gamma$ is the circle of radius $R$, centered at the origin, traced counterclockwise.
(b) Compute the scalar curl of $\mathbf{F}$, and show that it is 0 if and only if $\alpha = 1$.
(c) Take $\alpha = 1$. Let $\gamma$ be the circle, centered at a point $(x_0, y_0)$, with radius $R \neq \sqrt{x_0^2 + y_0^2}$, traced counter-clockwise. What is $\int_\gamma \mathbf{F}_\alpha \cdot \mathbf{dx}$? Your answer will depend on $(x_0, y_0)$ and $R$. Justify your answer.

(3) Find the general (real) solution of

$$y'' - 2y' + 10y = 0, \quad y(0) = 2,\ y'(0) = 3$$

What happens to the solution as $x \to \infty$?

(4) Find the general (real) solution of

$$y'' - 5y' + 4y = e^x$$

(5) Solve

$$y' + \sin(3x)y^2 = 0, \quad y(0) = 1$$

What is $y(\pi)$?