Math 341, Jeffrey Adams
Review Test I, March 14, 2011

1. Chapter 9: Vector Field Theory
   (a) Green's Theorem, independence of path, closed paths
   (b) Green's Theorem=Stokes's theorem in the plane
   (c) Gauss's Theorem in $\mathbb{R}^2$
   (d) Gauss's Theorem = Divergence Theorem in the plane
   (e) Conservative vector fields
   (f) Gradient vector fields
   (g) Simply connected regions
   (h) The main theorem in $\mathbb{R}^2$ (see below)
   (i) Surface integrals in $\mathbb{R}^3$, normal vectors, surface area
   (j) Oriented surfaces
   (k) Gauss's theorem in $\mathbb{R}^3$
   (l) Stoke's theorem in $\mathbb{R}^3$
   (m) $\nabla, \nabla\cdot,$ curl: formal properties
2. Chapter 10: First Order Differential Equations
   (a) Basics of first order differential equations; existence and uniqueness theorem
   (b) Separation of variables
   (c) Linear first order equations, exponential integrating factor
   (d) Applications ("word problems")
3. Chapter 11: Second Order Differential Equations
   (a) Constant coefficient, linear, homogeneous
   (b) distinct roots, complex roots, repeated roots
   (c) Higher order linear equations
   (d) Linear, nonhomogeneous equations, $y = y_p + y_h$
   (e) Undetermined coefficients
   (f) Variation of parameters
   (g) Oscillations

The main theorem of Chapter 9 is:

Theorem: Suppose $\mathbf{F}$ is a continuously differentiable vector field, defined in an open set B in $\mathbb{R}^2$ or $\mathbb{R}^3$. Consider the conditions:

1. $\mathbf{F}$ has independence of path;
2. $\int_\gamma \mathbf{F} \cdot \mathbf{dx} = \mathbf{0}$ for any closed path γ;
3. $\mathbf{F} = \nabla \mathbf{f}$ for some f
4. $\text{curl}(\mathbf{F}) = \mathbf{0}$ in B

Then $1 \Leftrightarrow 2 \Leftrightarrow 3 \Rightarrow 4$, and $4 \Rightarrow 1$ if B is simply connected.