Math 340, Jeffrey Adams

Test I, November 19, 2010

For complete credit you must show all work

Question 1 [15 points] Let $f(x,y) = \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix}$.

(a) $f'(x,y) = \begin{pmatrix} 2x & 2y \\ 2x & -2y \end{pmatrix}$

(b) The determinant is $-4y^2 - 4x^2 = -4(x^2+y^2)$. This equals 0 only if $x = 0, y = 0$, so the only point is $(0,0)$.

Question 2 [10]

(a) The gradient is $(2x, 2y, -4z)$, which at $(1,1,1)$ gives $\vec{v} = (2,2,-4)$. The gradient points in the direction of maximum increase; when you have a level set, **this is perpendicular to the surface**. If you take something perpendicular to $\vec{v}$ you are getting a **tangent vector**.

(b) The equation is $((x,y,z) - (1,1,1)) \cdot \vec{v} = 0$, which is $2(x-1) + 2(y-1) - 4(z-1) = 0$, which simplifies to $x + y - 2z = 0$.

Question 3 [20] The region is under a parabola, and over the x-axis. To find extremal points in the interior interior, $\nabla f(x,y) = (4x, 1+2y)$, which equals 0 only at $(0, -\frac{1}{2})$ which is not in the region. So the only max/min are on the boundary.

Now do two Lagrange interpolation problems, one for each component of the boundary.

Let $G(x,y) = x^2+y-1$. Then $\nabla f - \lambda\nabla G = (4x+2\lambda x, 1+2y+\lambda)$. Setting this equal to 0 gives $x = 0$, or $\lambda = -2$. If $x = 0$ then $y = 1$. Otherwise $\lambda = -2$ gives $y = \frac{1}{2}$, and then $x = \pm\frac{\sqrt{2}}{2}$.

If $y = 0$, just look at $f(x,y) = f(x,0) = 2x^2$, which has a minimum at $x = 0, y = 0$.

You should consider the "corners" separately, i.e. $(\pm 1, 0)$. So the points to check are: $(0,1), (\pm 1, 0), (\pm\frac{\sqrt{2}}{2}, \frac{1}{2})$.

Checking these points give $f(0,1) = f(\pm 1, 0) = 2$ is a max, and $f(0,0) = 0$ is a min.

Question 4 [15] This is an ellipse. $f'(t) = (-2\sin(t), 3\cos(t))$, and $|f'(t)| = \sqrt{4\sin^2(t) + 9\cos^2(t)}$, so the integral is $\int_0^{2\pi} \sqrt{4\sin^2(t)+9\cos^2(t)}$. (By the way the corresponding indefinite integral is an *elliptic integral* which cannot be evaluated in terms of elementary functions.

Question 5

The matrix equation is:

$$\begin{pmatrix} x_s & x_t \\ y_s & y_t \end{pmatrix} = -\begin{pmatrix} s^3 & -\sin(\pi(y+t)\pi \\ yst & xst \end{pmatrix}^{-1} \begin{pmatrix} 3xs^2 & -\sin(\pi(y+t))\pi \\ xyt & xys \end{pmatrix}$$

where you need to first plug in $(x, y, s, t) = (2, 1, 1, 1)$ in the matrices on the right, i.e.

$$\begin{pmatrix} x_s & x_t \\ y_s & y_t \end{pmatrix} = -\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 6 & 0 \\ 2 & 2 \end{pmatrix}$$

which gives

$$\begin{pmatrix} x_s & x_t \\ y_s & y_t \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 6 & 0 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 06 & 0 \\ 2 & 1 \end{pmatrix}$$

So $x_s = -6, y_s = 2, x_t = 0, y_t = -1.$

Question 6

(a) closed

(b) neither

(c) closed

(d) open