MATH 340 EXAM 2
NOVEMBER 17, 2009

1.(15) A bug is crawling on a piece of heated graph paper and notices that if it moves in the positive x-direction it heats up by $2\frac{\text{deg}}{\text{cm}}$ and if it moves in the positive y-direction it cools down by $3\frac{\text{deg}}{\text{cm}}$. In which direction should it move to a) heat up most rapidly b) cool down most rapidly and c) remain at the same temperature.

2.(20) a) Find and classify all critical points (max, min or saddle point) of the function $f(x, y) = \frac{1}{2}x^2 - xy + y^3$.

b) Explain why $f(x, y) = x^2 + y^2 + x$ has a maximum and minimum value on the elliptical region $2x^2 + y^2 \leq 4$ and find the minimum and maximum values.

3.(15) Show you can solve the equations

$$\begin{aligned} vy - u\cos x &= 3 \\ v\sin x + yu &= 2 \end{aligned}$$

for (x, y) as functions of (u, v) near the point $(u, v, x, y) = (2, 1, \pi, 1)$ and determine $\frac{\partial u}{\partial x}(\pi, 1)$.

4.(15) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable and $g : \mathbb{R}^3 \to \mathbb{R}^2$ is given by $g(x, y, z) = (\frac{x}{y}, \frac{z}{y})$. Let $F = f \circ g : \mathbb{R}^3 \to \mathbb{R}$, i.e. $F(x, y, z) = f(g(x, y, z)) = f(\frac{x}{y}, \frac{z}{y})$. Show

$$x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} + z\frac{\partial F}{\partial z} = 0.$$

5.(20) a) Find the volume of the region in $\mathbb{R}^3$ bounded below by the paraboloid $z = x^2 + y^2$ and above by the cone $z^2 = x^2 + y^2$, $z \geq 0$.

b) Use the change of variables $x = u + v$, $y = \frac{v}{u+v}$ to evaluate the integral $\int_D x\,dV$ where D is the region in $\mathbb{R}^2$ bounded by $xy = 1$, $xy = 2$, $x(1 - y) = 1$ and $x(1 - y) = 2$.

6.(15) Consider the vector field $\mathbf{F}(x, y) = (xy, y^3)$ in $\mathbb{R}^2$ and the following two curves from $(0, 0)$ to $(1, 1)$. C_1 is the straight line between the two points and C_2 is the path which follows the parabola $y = x^2$. Compute $\int_{C_1} \mathbf{F} \cdot d\mathbf{x}$ and $\int_{C_2} \mathbf{F} \cdot d\mathbf{x}$. Is $\mathbf{F}$ a gradient field?