Math 340, Jeffrey Adams
Test I, November 19, 2010
For complete credit you must show all work

Question 1 [15 points] Let $f(x,y) = \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix}$.

(a) Compute $f'(x,y)$

(b) For what values of x, y is the function f *not* invertible near (x,y).

Question 2 [10] Consider the surface S in $\mathbb{R}^3$ consisting of (x,y,z) such that $x^2 + y^2 - 2z^2 = 0$.

(a) Find a vector perpendicular to S at the point $(1,1,1)$.

(b) Find an equation of the tangent plane to S at this point.

Question 3 [20] Find all maxima and minima of the function $f(x,y) = 2x^2 + y + y^2$ on the set of (x,y) such that $0 \leq y \leq 1 - x^2$.

Question 4 [15] Consider $f(t) = (2\cos(t), 3\sin(t))$. Describe the parametrized curve this defines. Write down an integral to compute the length of the curve. It is *not* necessary to compute the integral.

Question 5 [20] Suppose x, y, s, t are related by

$$\begin{aligned} xs^3 + \cos(\pi(y+t)) &= 3 \\ xyst &= 2 \end{aligned}$$

and this defines x, y implicitly in terms of s, t.

(a) Find $\frac{\partial x}{\partial s}$ and $\frac{\partial y}{\partial s}$ at $(x,y,s,t) = (2,1,1,1)$

(b) Find $\frac{\partial x}{\partial t}$ and $\frac{\partial y}{\partial t}$ at $(x,y,s,t) = (2,1,1,1)$

Question 6 [20] For each of the following subsets of $\mathbb{R}^2$, state whether it is open, closed, both, or neither.

(a) S is the set of (x,y) such that $x^2 + y^2 \leq 1$ *or* $(x-1)^2 + y^2 \leq 1$

(b) S is the set of $(x,0)$ with $0 < x < 1$

(c) S is the graph of the function $y = x^2$, i.e. the set of points (x, x^2) for $x \in \mathbb{R}$

(d) S is the set of points (x,y) with $0 < x < 1$ and $0 < y < 1$