Math 340, Jeffrey Adams

Test I, October 11, 2010

For complete credit you must show all work

(1) [12 points] Compute:

(a) $(1,2,3) \cdot (4,5,6)$

(b) $(1,1,1) \times (1,0,-1)$

(c) $|-2\vec{v}|$ if $|\vec{v}| = 5$

(d) A unit vector $\vec{n}$ which is a multiple of $(-1,2,-3)$

(2) [20] Let $\vec{v} = (1,1,1)$ and $\vec{w} = (1,0,2)$ and let P be the plane spanned by $\vec{v}$ and $\vec{w}$.

(a) Find a vector $\vec{v}$ perpendicular to P.

(b) Find an equation for P, of the form $ax + by + cz = 0$ for some a, b, c.

(c) Let $\vec{w} = (2,3,5)$, and set $P' = \{\vec{w} + \vec{u} \,|\, \vec{u} \in P\}$, i.e. the translate of P by $\vec{w}$. Find an equation for P'.

(3) [15] For what value(s) of a is the matrix $\begin{pmatrix} 1 & 1 & 0 \\ a & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ *not* invertible? For such an a, what is the dimension of the nullspace of the matrix?

(4) [20] Consider the equations

$$2x - y = -1$$
$$x - y + 2z = 0$$

(a) Find all solutions to the associated homogeneous equation

$$2x - y = 0$$
$$x - y + 2z = 0$$

(b) Find a single solution to the original equation.

(c) Find all solutions to the original equation.

(5) [15] Find all eigenvalues of $M = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$. For each eigenvalue find a basis of the corresponding eigenspace.

(6) [18] Suppose f is an *injective* linear function from V to W and $\{v_1, \dots, v_n\}$ is a basis of V. Let $S = \{f(v_1), \dots, f(v_n)\} \subset W$.

1. Show that S is a linearly independent set in W.
2. Show that S is a basis of W if $\dim(V) = \dim(W)$.
3. Show that S is not a basis of W if $\dim(V) < \dim(W)$.