MATH 340 FINAL EXAM
DECEMBER 17, 2009

1.(20) True or false. Give a <u>brief</u> reason.

a) If A is an $m \times n$ matrix with $m < n$ then the system of equations $A\mathbf{x} = \mathbf{0}$ always has a non-trivial solution.

b) The determinant of any 11×11 matrix with $A = -A^t$ is zero. (A^t is the transpose of A.)

c) If P_n is the space of polynomials of degree $\leq n$ then the function $T : P_n \to \mathbb{R}^2$ with $T(p) = (p'(0), p(0)^2)$ is a linear map.

d) If A is a 12×20 matrix and and for every $\mathbf{y} \in \mathbb{R}^{12}$ there exists $\mathbf{x} \in \mathbb{R}^{20}$ with $A\mathbf{x} = \mathbf{y}$ then there exists 8 linearly independent solutions to the equation $A\mathbf{x} = \mathbf{0}$.

e) The linear map $R : \mathbb{R}^2 \to \mathbb{R}^2$ given by rotation about the origin by $\frac{\pi}{2}$ in the counterclockwise direction has matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

2.(10) Let $f(x, y) = (\cos x + \sin y + x, x^2 + 2y)$. Use the tangent approximation to find (x, y) so that $f(x, y) \sim (.98, .01)$. (Note: $f(0, 0) = (1, 0)$.)

3.(10) A bug is moving in $\mathbb{R}^3$ with position $\mathbf{x}(t) = (4t, \sqrt{3}\, t^2, t^3 + 3t)$ for $t \geq 0$.

a) How far has the bug travelled (along the path) after 1 second.

b) If $T(x, y, z) = x^2 + 2y^2 - z^2$ is the temperature at a point (x, y, z) is the bug getting warmer or cooler at $t = 1$ if he continues along the path?

4.(5) Let Σ be intersection of the ellipsoid $2x^2 + y^2 + z^2 = 2$ and the smooth surface $\sin(xy) + yz = 1$. Explain why in a neighborhood of $(0, 1, 1) \in \Sigma$, (x, y) may be written as functions of z but it is not clear if (y, z) may be expressed in terms of x.

5.(5) Explain why if $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a differentiable function and $f(0, 0) = (1, 1)$ and $Df(0, 0) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ then there does not exist a differentiable function $g : \mathbb{R}^2 \to \mathbb{R}^2$ with $(f \circ g)(x, y) = (y, x)$ and $g(1, 1) = (0, 0)$.

6.(10) Compute the following integrals: a) $\int_0^\pi \int_y^\pi \frac{\sin(x)}{x}\, dx\, dy$.

b) $\int_R (x^2 - y^2)\, dA$ where R is the region in the xy-plane bounded by the lines $x + y = 1$ and $x + y = 2$, $y = x$ and the y-axis.

7.(10) Let C be a closed curve in $\mathbb{R}^2$. Use Green's theorem to show the two vector fields $\mathbf{F}_1(x, y) = (0, x)$ and $\mathbf{F}_2(x, y) = (y, 0)$ do opposite work around the closed curve C.

8.(10) Find the surface area of that part of the paraboloid $z = \frac{1}{2}(x^2+y^2)$ cut out by the cylinder $x^2 + y^2 = 4$.

9.(10) Let $\mathbf{F}(x, y, z) = (0, 0, z^2)$ and let D be the solid region in $\mathbb{R}^3$ given by $\{(x, y, z) \,|\, 0 \leq x^2 + y^2 \leq 1, 0 \leq z \leq 1\}$. Verify Gauss's Theorem by computing both sides of the equation.

10.(10) Let $\mathbf{F}(x, y, z) = (y^2, x^2+z, -xy)$. Compute line integral $\int_\Sigma \mathbf{F} d\mathbf{x}$ where Σ is the intersection of the paraboloid $z = (x - 1)^2 + (y - 1)^2$ and the plane $2x + 2y + z = 6$ oriented counterclockwise when viewed from above.