Phys 761 "Making and sustaining a Plasma" 9 November 2010

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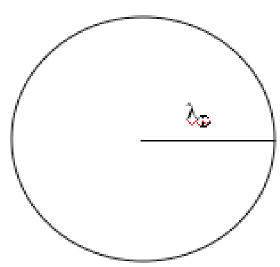
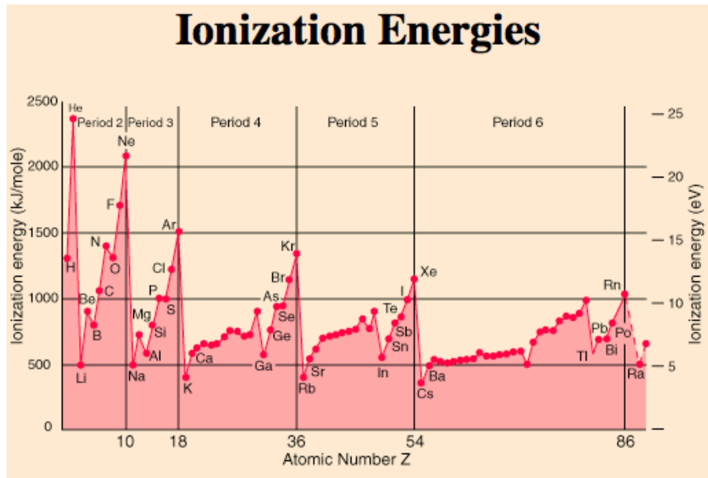
- We function in a relatively low temperature and high density environment so most of the matter around us is in a molecular state.
- To make a plasma and sustain it we need to ionize the material and to keep it from reverting back to the molecular state.
- To do this at liquid or solid densities requires very large power inputs to the material we want to make into a plasma. You can do it by initiating a nuclear explosion or, in small volumes, by irradiating the material with very high power lasers.
- It is much easier to make a plasma from a gaseous material as you can lower the density with vacuum pumps and so lower the power requirements.
- The external electric field that will do the work to ionize the atoms can be produced by an incident charged particle or a photon.
- To make a plasma we must boost the electrons out of the electrostatic wells in which they are confined in the atoms and get a few of them into a Debye sphere. Some of the atoms in this room are ionized by the radioactive nuclei in the materials around us and incident cosmic rays, but we are not living inside a plasma. The Debye shere has a radius equal to a Debye length. On the diagran is an expression fro calculating the number of particles in a Debye sphere.

# of electrons in a Debye Sphere

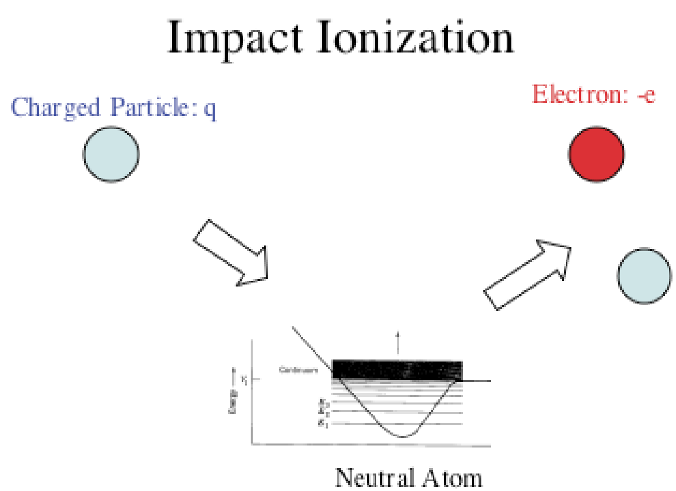


$$N_D = 1.72\text{x}10^{12}\,\frac{T_e^{3/2}(\text{eV})}{n_e^{1/2}(\text{m}^{-3})}$$

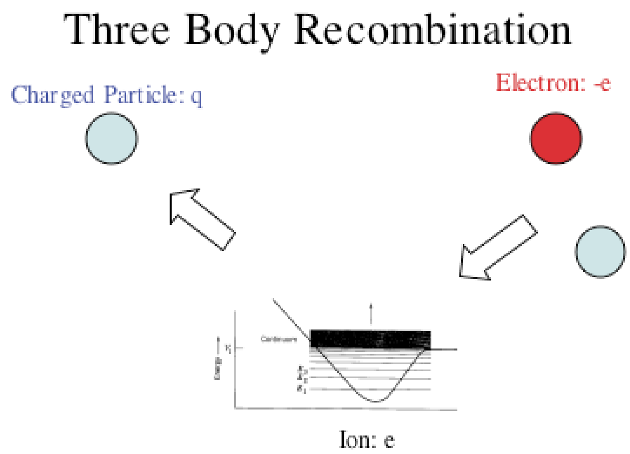
How much energy do we need for ionization? The diagram shows the energy required to ionize the elements of the periodic table.



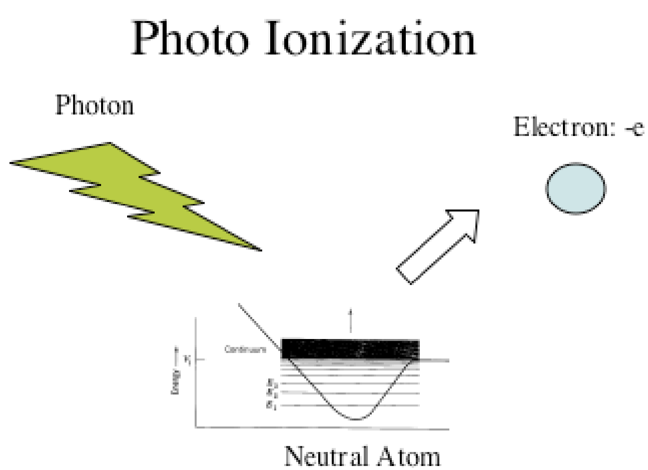
- Impact Ionization. Notice the internal structure of the atom. A charged particle collides with an atom, an electron is ejected and the charged particle and the ejected electron depart.



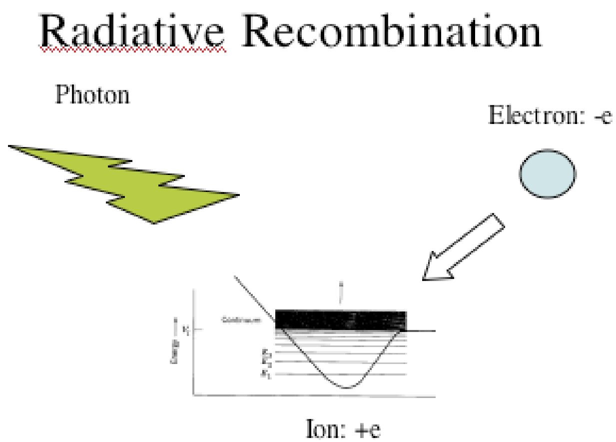
- Three Body Recombination. The inverse process for impact ionization. In this inverse process, an electron and another charged particle collide with an ion, the electron is capture by the ion and thereby neutralized, and the other charged particle leaves carry off the energy and momentum required by the conservation laws.



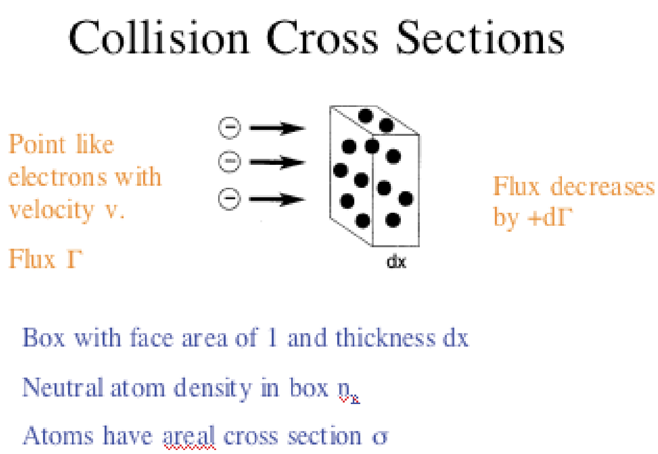
- Photo Ionization. Here an incoming photo collides with the atom, is absorbed and an electron is ejected.



- Radiative Recombination. The inverse process of photo ionization. In this process which is the inverse of photo ionization, an incoming electron is captured by an ion, and a photon is emitted and escapes.



- Collision Cross Sections. This diagram reproduces Figure 10.2 from Goldston and Rutherford.



- The flux on the left side of the box is Γ.
- The flux on the right side of the box is $\Gamma + d\Gamma$
- The incoming flux is reduced by the atoms in the box by an amount equal to the area that they obstruct. So

$$\Gamma + d\Gamma = \Gamma\left(1 - \sigma n_n \{1dx\}\right)$$
$$1 = \text{ cross sectional area of box}$$
$$1dx \equiv \text{ volume of box}$$
$$n_n \equiv \text{ density of atoms}$$
$$\sigma \equiv \text{ cross section area of atom}$$

So

$$\frac{d\Gamma}{\Gamma} = -\sigma n_n dx$$

Integrate $\left[\ln\Gamma\right]_{\Gamma_0}^{\Gamma} = -\sigma n_n \left[x\right]_0^x$

Take antilog $\dfrac{\Gamma}{\Gamma_0} = e^{-\sigma n_n x} = e^{-\frac{x}{\lambda_{mfp}}}$ Definition of mean free path.

where $\lambda_{mfp} = \dfrac{1}{\sigma n_n}$

Time between collisions. $\tau = \dfrac{\lambda_{mfp}}{v} = \dfrac{1}{\sigma n_n v}$

Frequency of collisions. $\nu = \dfrac{1}{\tau} = \sigma n_n v$

This is a cartoon version of reality. It really is a quantum mechanical process with an electron with a DeBroglie wavelength impacting on a quantum mechanical atom. This means that the cross sectional area is function of the electron velocity (its wavelength). So we need to use a velocity weighted cross section. This is usually done for a Maxwellian distribution of electron velocities.

- The Source Rate for ionization, the rate at which electrons are produce per unit volume is given by:

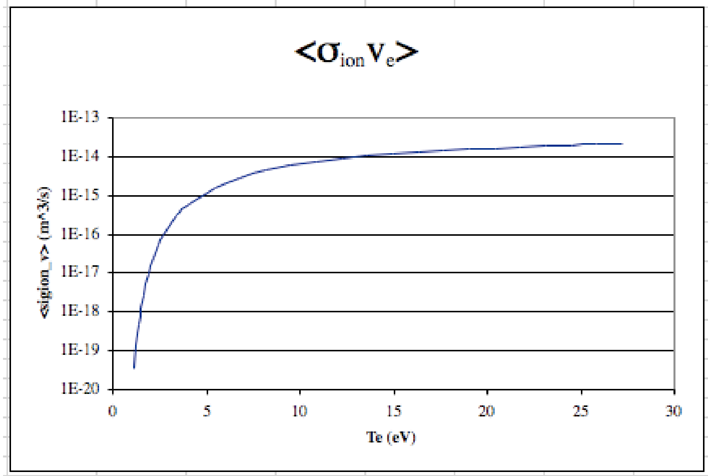
$$S_e = n_e n_n \langle \sigma_{ion} v_e \rangle$$

- We assume that the electrons initially released rapidly collide with each other to produce a Maxwellian distribution of velocities. That is they thermalize and the then carry on the ionization process.
- The ionization rate for atomic hydrogen is:

$$\langle \sigma_{ion} v_e \rangle = \frac{2x10^{-13}}{6 + \overline{T}_e} \overline{T}_e^{\frac{1}{2}} e^{-\frac{1}{\overline{T}_e}} \quad m^3 s^{-1}$$

$$\overline{T}_e = \frac{T_e(eV)}{13.6}$$

The diagram below is similar to Figure 10.4 in Goldston and Rutherford



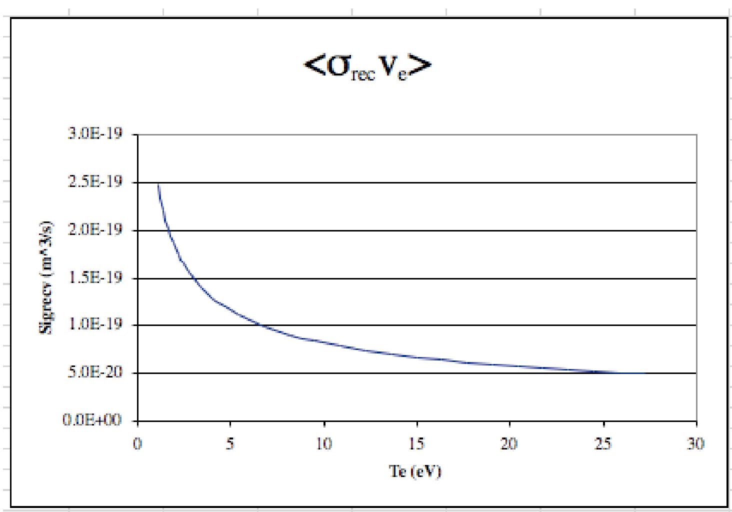
- The sink rate for recombination, the rate at which electrons are remove from the plasma per unit volume is given by:

$$S_n = n_e n_i \langle \sigma_{rec} v_e \rangle$$

- The radiative recombination rate is:

$$\langle \sigma_{rec} v_e \rangle = 7x10^{-20} \left(\frac{1}{T_e} \right)^{\frac{1}{2}} \quad m^3 s^{-1}$$

This quantity is plotted in the diagram below.



The Coronal Equilibrium is established by setting the Source rate equal to the radiative recombination Sink rate.

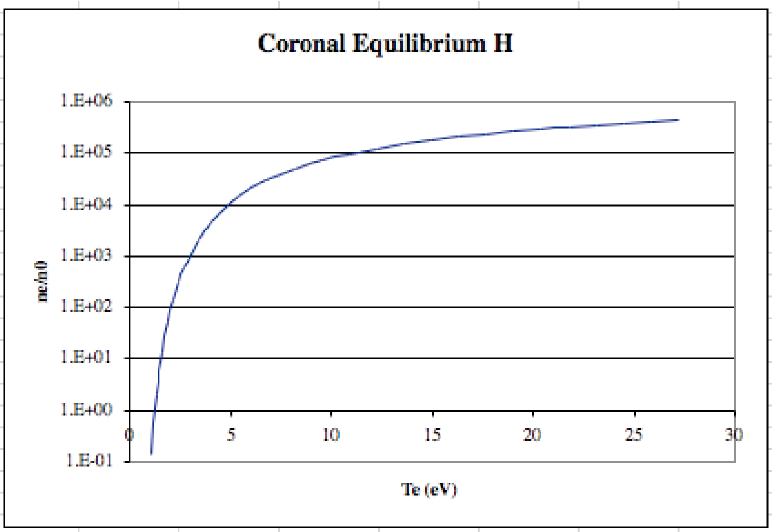
$$S_e = n_e n_n \langle \sigma_{ion} v_e \rangle = S_n = n_e n_i \langle \sigma_{rec} v_e \rangle$$

$$\frac{n_i}{n_n} = \frac{\langle \sigma_{ion} v_e \rangle}{\langle \sigma_{rec} v_e \rangle}$$

$$\frac{n_i}{n_n} = \frac{\dfrac{2x10^{-13}}{6+\overline{T}_e}\,\overline{T}_e^{\frac{1}{2}} e^{-\frac{1}{T_e}}}{7x10^{-20}\left(\dfrac{1}{\overline{T}_e}\right)^{\frac{1}{2}}}$$

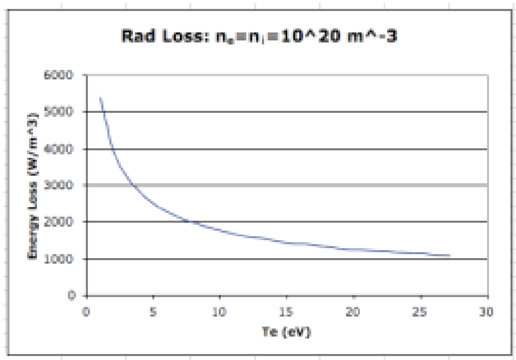
$$\frac{n_i}{n_n} = \frac{\frac{2}{7}x10^{7}}{6+\overline{T}_e}\,\overline{T}_e e^{-\frac{1}{T_e}}$$

The diagram below is similar to Figure 10.5 in Goldston and Rutherford.



Now if you have created a hydrogen plasma in such a coronal equilibrium, each time an ion recombines, it emits an ultraviolet photon which escapes and its energy is lost from the plasma. To get back to the equilibrium another ionization has to take place to replace the electron that was lost in the recombination. To do that the plasma electrons have to supply this ionization energy. Now they are short of this amount of energy, which must be replaced from outside the plasma if the equilibrium is to be sustained. We can estimate the amount of energy that has been lost and the rate that it must be replaced to sustain the plasma, by multiplying the “sink” rate by the ionization energy to get the number of watts needed per unit volume. The results of doing this are shown in the diagram below.

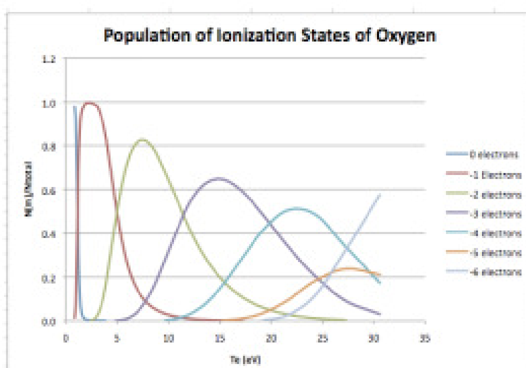
Power Losses in Pure Hydrogen Plasma



These power values are quite modest, but, unfortunately, they represent a Utopian situation. It is not possible to make a pure plasma and usually plasmas are contaminated with carbon, oxygen and other elements. We will look at oxygen as an impurity and examine the consequences for sustaining of an impure plasma.

If oxygen atoms are in a plasma they will be ionized and as each atom has 8 electrons, the electrons can possibly be removed one at a time. Within a plasma of a specified electron temperature there will be a population of oxygen ions with various degrees of ionization, i.e. with various numbers of electrons removed from the ion. This is illustrated in the diagram below.

Population of Oxygen Ions

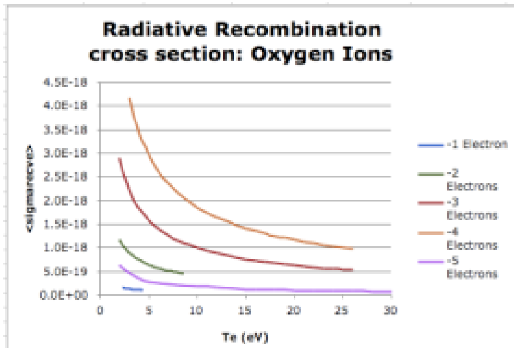


J. W . Allen and A. K. Dupree: Astrophysical Journal **155 p** 27 (Jan 1969)

Note that at Te = 2.2 eV nearly 100% of the oxygen atoms have had one electron removed. At Te = 7.7 eV about 80% of the ions have had 2 electrons removed. At Te =15.3 eV about 65% of the ions have had 3 electrons removed. At Te = 21.7 eV about 50% of the ions have had 4 electrons removed.

The radiative recombination cross section for oxygen increases as more electrons are removed. When 4 electrons are removed it the cross section is about an order of magnitude larger than that for hydrogen. See the diagram below.

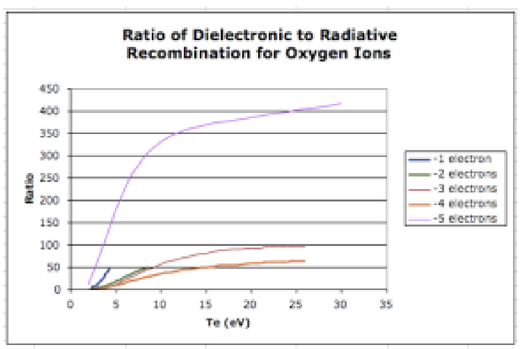
Radiative Recombination Rate for Oxygen



S. M. V. Aldrovandi and D. Pequignot: Astron. & Astrophys. **25**, 137-140 (1973)

Unfortunately the three body recombination rate, also called dielectronic recombination, becomes large as the number of electrons removed increases. The ratio of the dielectronic rate to the radiative rate is shown below.

Ratio of Dielectronic to Radiative Recombination Rates for Oxygen



S. M. V. Aldrovandi and D. Pequignot: Astron. & Astrophys. **25**, 137-140 (1973)

As you can see from the diagram the dielectronic rate dominates. When an electron is captured by this process it returns to the lowest energy state available by emitting a photon. For the higher states of ionization these photons are much more energetic than the ones emitted in hydrogen recombination. When the first ionized electron is recombined, the photon energy is 13.6 eV, for the second 35.1 eV, for the third 55 eV and for the fourth 77eV. The result of this greatly enhanced recombination rate and the increased photon energy is a radiation barrier to higher plasma temperature. If more power is put into the plasma, the temperature does not increase. The impurity ions radiate away the power that is input to the plasma. If a hydrogen plasma has too many higher z impurity ions, it is very difficult to increase the plasma temperature above about

30 eV. The radiated power becomes of the order megawatts per cubic meter. And there are several other ways for the plasma to lose energy, so sustaining high temperature plasmas is a substantial challenge.

However high temperature hydrogen plasmas are produced by working hard to keep the impurity fraction small. Plasmas of density about 10^{20} m^{-3} and plasma temperature about 10 keV are produced with power inputs of about 1 MW per cubic meter.