Phys604/F15/Hassam/Problem Set 9 Due 11/10/15

9.1

Try a Frobenius series type solution for Legendre functions* in powers of $s=(1+x)$. Start from the complete Legendre equation for arbitrary ν. Rewrite this equation in the s variable and then try the series solution.

(a) Show that only one solution is obtainable. Check that this is consistent with the asymptotic behaviour of Legendre functions as $s \to 0$ (or $x \to -1$).

(b) Show that the one good series solution may diverge at $x=1$ (a simple ratio test suffices). (In fact, this solution does, in general, diverge at $x=1$.)

(c) Show that the series terminates if $\nu = n =$ integer. Thus, for integer ν, one solution is well-behaved at both $x=\pm 1$. Find these integer solutions for n=0, 1,and 2, rewriting them in the x variable.

(d) *Compare (just browse) what you have found with what is asked for in AWH (7th Ed) Chapter 8 Problem 3.1.

9.2

(a) Try the Frobenius method to find solutions if y(x) satisfies the ODE $y'' = y/x^4$. Assume a_0 to be non-zero in commencing your trial solution. How many solutions can be found for this form of a series expansion?

(b) Find the leading order asymptotic behavior of this equation as $x \to 0$. Note that the Laurent expansions of the asymptotic solutions about $x = 0$ have a radius of convergence which does not include $x = 0$; and, in any case, the Laurent expansion does not have the same form as the starting ansatz of the Frobenius series.

9.3

The function $\phi(x)$ satisfies

$(d/dx)[x d\phi/dx] - \phi/x = 0$, $\phi(1) = 1$, $\phi(\infty) = 0$. Note that the domain excludes x=0, hence division by x is ok.

1.1 Find the solution by directly solving the ODE

1.2 Find the Green function for this problem, sketch a plot vs x'

1.3 Find $\phi(x)$ using the Green function and compare

[1D Green function notes are posted.]