**Phys604/F15/Hassam/Problem Set 8 Due 11/03/15**

8.1 Find the asymptotic behaviour of the solutions to the Airy equation $y'' = xy$ for $x \to 0$. Keep terms upto $x^5$ in the expansion. Compare your asymptotic expansion for $|x| << 1$ to the Taylor expanded solution from the exact integral representation of the Airy Function that was obtained in class. (See posted notes.) In particular, show:

(1) that the coefficients of the $x^2$ and the $x^5$ terms are identically zero, consistent with your asymptotic series;
(2) that the ratio of the coefficients of the $x^3$ and the $x^0$ terms are consistent with your asymptotic expansion. Do likewise for the $x^4$ and the $x^1$ terms.

8.2 Legendre functions satisfy

$(d/dx)[(1 - x^2)(dy/dx)] + \nu(\nu+1)y = 0$, for arbitrary $\nu$.

Assume $\nu \sim O(1)$. Obtain the asymptotic behavior of the Legendre functions near the singular points $x = \pm 1$. To scale appropriately near $x \to +1$, first evaluate the differential operator in the variable $s = (x - 1)$, for small s. Keep the first two terms in this operator. Then, scale and find a perturbation solution upto two terms. Do likewise for $x \to -1$.

Next, find the asymptotic behaviour of the Legendre functions functions, to leading order, for positive x and $x >> 1$.

8.3 Find the asymptotic behaviour, to leading order, of the solutions to $xy'' = y$, for positive $x >> 1$. Do this by directly solving the equation (asymptotically, correct to first order) and also by starting from the integral solutions obtained in the previous problem set. Show agreement between the two approaches.