Phys604/F15/Hassam/Problem Set 4 Due 10/06/15

Arfken, Weber, Harris, 7^{th} Ed
Chapter 11:

11.6.11

11.7.1 [(a), (c), (e), (h)] (you may select to evaluate one pole for each example)

8.9, 8.12(b), 8.15

Other

4.1 Consider a contour C which is an arc of a circle of radius R, going from $z1=R$ to $z2=R\exp(i\phi)$. Evaluate exactly the integral $I = \int dz \exp[-az]$ along C. Let a be real and positive. Show that $|I| \to 0$ as $R \to \infty$, but only for a specific range of ϕ. Specify this range.

4.2 Consider the function (discussed in class) $F(z) = \int dt \exp(-t^2) / (t-z)$, where $z = x + iy$ and $y > 0$. The analytic continuation of F(z) for $y \leq 0$ can be obtained by deforming the original contour to below the real axis which then allows z to be moved to below the real axis but above the contour. Show that the analytic continuation of F(z) for z=x is

$$F(x) = P\int dt \exp(-t^2) / (t-x) \quad + \quad i\pi \exp(-x^2),$$

where P denotes Cauchy Principal Value wherein the integral over t runs from $-\infty$ to $(x - \varepsilon)$ and from $(x + \varepsilon)$ to $+\infty$, as $\varepsilon \to 0$.

4.3 Evaluate $I = \int_C dz\, f(z)$, where $f(z) = [(z^2+1)(z-2i)]^{-1}$, and C runs along the x-axis from $-\infty$ to $+\infty$. Do this in two ways: deforming C upwards and then downwards.