**Phys604/F15/Hassam/Problem Set 2 Due 9/22/14**

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Chapter 11: 2.3(a), 2.7

Chapter 11: 3.6, 3.7 (modified as below), 4.2

3.7 (modified as follows):
State clearly why Cauchy's Integral Theorem cannot be used. Then, do the integral using the following two methods:

1. The contour obtained by stretching the given contour to a $|z| \to \infty$ circle is an equivalent contour. Why? Use this information and the simple integrals explicitly done in class (see also 2.3 below) to do your integral.

2. Use partial fractions, split the problem into 2 separate integrals, then use Cauchy's Theorem as well as the simple integrals from class (or 2.3).

Other

2.1 Let $f(x,y) = xy^2(x+iy)/(x^2+y^2)$ for all $z \neq 0$ with $f(0,0) = 0$. Determine where if anywhere is this function (a) differentiable, ie, df/dz exists; and (b) analytic.

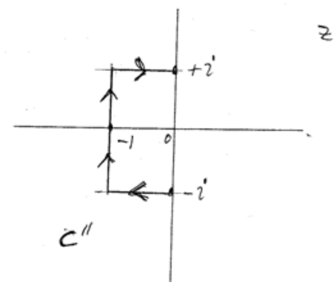
2.2 Consider $f(z) = 1/z^{1/2}$. Place a branch cut along $\theta = \pi$.

1. Plot the real and imaginary values of f(z) along the unit circle contour, starting from $\theta = -\pi/2$ and going clockwise to $\theta = +\pi/2$. Call this contour C. Note clearly any discontinuities along the path.

2. By *direct contour integration*, find the value of the definite integral $\int dz\, f(z)$ along the contour C as above. State why integrating over the discontinuities is not an issue.

3. Consider the unit circle contour, starting from $\theta = -\pi/2$ and going counter-clockwise to $\theta = +\pi/2$. Call this contour C'. Find the value of the definite integral $\int dz\, f(z)$ along C' by using anti-derivatives. Why is there (obviously) no path independence for the integrals between C and C'?

4. Consider the contour C'' as shown in the Figure. Can you find the value of ∫ dz f(z) along this contour by using only the info from the two integrals already done above? Justify. (use a limiting process)




<u>2.3</u> Consider the integral ∫ dz $z^n$, n=integer, taken counter-clockwise along the unit circle about the origin. You are asked to do this 2 different ways for all n.

(a) Do it by brute force direct contour integration, as was done in class.

(b) Introduce a cut-line as appropriate and then do the integral using Cauchy's Theorem, ie, by antiderivatives (appropriately) and proceeding to the limit of a full circle.

<u>2.4</u> Show that if a function f(z) satisfies the CR conditions, then the function must be differentiable. [This is the reverse proof of the CR conditions; you may assume continuity and smoothness of the derivatives of u(x,y) and v(x,y).]