Phys604/F15/Hassam/Problem Set 1 **Due 9/15/14**

From Arfken/Weber/Harris (7th Ed)

Chapter 1: 1.8.6(a)

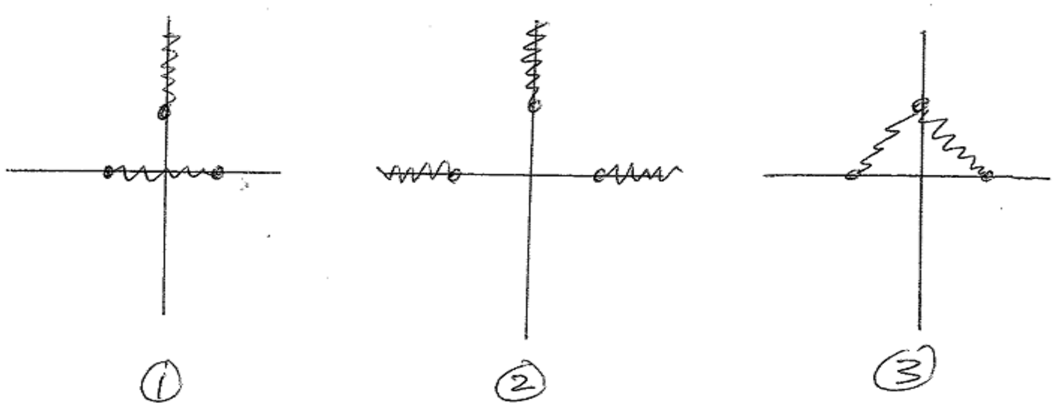
Other

1.1 Find all the zeroes of (a) sin(z) (b) sinh(z)

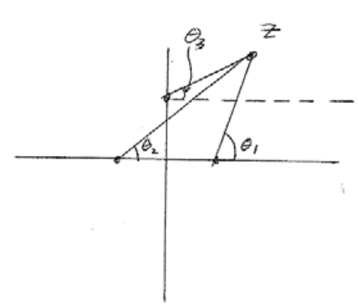
1.2 (a) Show that exp[ln(z)] always equals z; (b) Show that ln[exp(z)] does not always equal z

1.3 Where would you put branch cuts to make single-valued the function $f(z) = \ln(z^2 - 1)$?

1.4 Suppose $f(z) = (z^2-1)^{1/3} (z-i)^{1/3}$. Consider each of three possible placements for branch cuts shown below. Identify which of these give a single valued f(z) and why.



For Case 3, calculate the phase of f(z) at the two points $i(1+\varepsilon)$ and $i(1-\varepsilon)$, $\varepsilon\to 0$. Use the angles θ_1, θ_2, θ_3 as defined in the figure.



1.5 Suppose $f(z) = (z^2-1)^{1/3}$. Place a branch cut along the real axis from z=-1 to z=+1. Show that the resulting function is not single valued. Show one example of branch cuts that does yield a single valued function.

More from Arfken/Weber/Harris (7th Ed)

Chapter 11: 11.2.12 (adapted as follows):

Consider the general function $f = f(z,z^*) = u(x,y) + i\, v(x,y)$, where u and v are real functions, and consider the "coordinate transformations" between $(x,y) \leftrightarrow (z,z^*)$. Show that f is independent of z^*, ie, $(\partial f/\partial z^*)_z = 0$, $\Leftrightarrow$ f is analytic, ie, the CR conditions are satisfied. Your proof should work both ways. Use the chain rule for partials.