7.1H

$$V(x) = |x|\, V'(t) \quad , \quad V'(t) > 0$$

$$L = \frac{1}{2} m \dot{x}^2 - |x|\, V'(t)$$

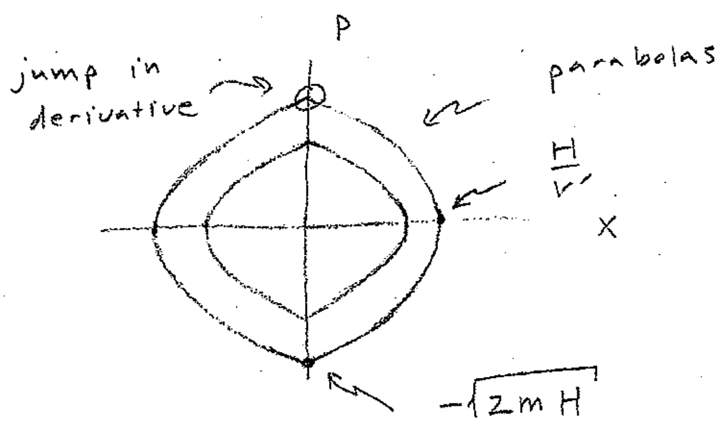
$$p = m \dot{x} \quad \rightarrow \quad H = p\dot{x} - L(x, p, t)$$

$$= \frac{p^2}{2m} + |x|\, V'(t)$$

EqO'M

$$\dot{x} = \frac{p}{m} \quad , \quad -\dot{p} = \frac{x}{|x|} V'(t)$$

$$V'(t) = \text{const.} \qquad |x| = \frac{H}{V'} - \frac{p^2}{2mV'}$$



7.2H

$$\vec{B} = [B_x(\vec{r}), B_y(\vec{r}), B_0]$$

let $\vec{B} = \hat{z} B_0 + \vec{\nabla} \times (\hat{z} A_z)$. Note $\vec{\nabla} \cdot \vec{B} = 0$

$$A_z = A_z(x, y, z) \ldots$$

Now $d\vec{r} \times \vec{B} = 0 \Rightarrow dx B_y = dy B_x$

$$dy B_z = dz B_y$$

$$dx B_z = dz B_x$$

$$\Rightarrow \frac{dx}{dz} = \frac{B_x}{B_0}, \quad \frac{dy}{dz} = \frac{B_y}{B_0}$$

$$B_x = \partial_y A_z, \quad B_y = -\partial_x A_z$$

$$\Rightarrow \boxed{\frac{dx}{dz} = \partial_y \psi, \quad \frac{dy}{dz} = -\partial_x \psi}$$

$$\boxed{\psi = A_z(x, y, z)/B_0}$$

$\psi(x, y, z) \longleftrightarrow \{x, y\}$ conjugates

H system

(7-34)

① $L = \frac{1}{2}m(\dot{x}^2+\dot{y}^2) - mgy \;,\; y = -x^2/2a$

$\dot{y} = -x\dot{x}/a$

$L = \frac{1}{2}m(\dot{x}^2 + x^2\dot{x}^2/a^2) + mgx^2/2a$

$L = L(\dot{x}, x)$

$\partial L/\partial\dot{x} = p = m(\dot{x} + \dot{x}\,x^2/a^2)$

$\Rightarrow p = m\dot{x}(1+x^2/a^2)$

$H = p\dot{x} - L$

$= m\dot{x}^2(1+x^2/a^2) - \frac{1}{2}m\dot{x}^2(1+x^2/a^2) - mgx^2/2a$

$$\boxed{H = \frac{p^2}{2m(1+x^2/a^2)} - mgx^2/2a} \quad \text{—①}$$

② $a=1, m=1, g=1 \Rightarrow$

$$\boxed{H = \frac{1}{2}\frac{p^2}{(1+x^2)} - x^2/2}$$

Thus $[L] \to a$, $t \to \sqrt{a/g}$, $\mathcal{E} \to mga$

Check this: From ①,

$$\frac{H}{mga} = \frac{1}{2}\frac{\dot{x}^2}{ga}\left(1+x^2/a^2\right) - x^2/2a^2$$

let $\frac{H}{mga} \to H$, $\frac{x}{a} \to x$

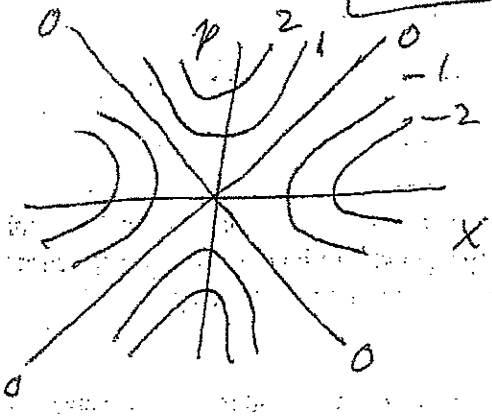
$$\Rightarrow H = \frac{1}{2}\frac{a^2\dot{x}^2}{ga}\left(1+x^2\right) - x^2/2$$

Now let $t^2\frac{g}{a} \to t^2$

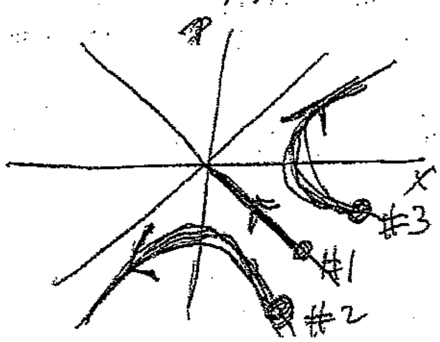
$$\Rightarrow \boxed{H = \frac{1}{2}\dot{x}^2(1+x^2) - x^2/2 = \frac{p^2}{2(1+x^2)} - \frac{x^2}{2}}$$

③ $|x| \ll 1 \Rightarrow \boxed{H \simeq \frac{p^2}{2} - \frac{1}{2}x^2}$ $\boxed{p \simeq \dot{x}}$

④



⑤ Use appx H here. $[x, p] = [1,-1], [1,-\sqrt{2}], [\sqrt{2},-1]$



$[1,-1] \Rightarrow H = 0$ #1

$[1,-\sqrt{2}] \Rightarrow H = 1/2$ #2

$[\sqrt{2},-1] \Rightarrow H = -1/2$ #3

⑥ $\dot{x} = \partial H/\partial p = p, \quad \dot{p} = -\partial H/\partial x = x$

$\dot{x} = p, \quad \dot{p} = x$

$\ddot{x} = x \Rightarrow x \sim \begin{Bmatrix} \sinh t \\ \cosh t \end{Bmatrix}$

let $x(t) = A \sinh t + B \cosh t$

$\Rightarrow p(t) = A \cosh t + B \sinh t$

#1 $\Rightarrow 1 = B, \; -1 = A$

$\Rightarrow \boxed{x_1 = -\sinh t + \cosh t}$

#2 $1 = B, \; -\sqrt{2} = A$

$\boxed{x_2 = -\sqrt{2} \cosh t + \sinh t}$

#3 $\sqrt{2} = B, \; -1 = A$

$\boxed{x_3 = -\cosh t + \sqrt{2} \sinh t}$

$T(x=0) \Rightarrow \boxed{\tanh T = -B/A}$

#1 $\tanh T = 1 \Rightarrow T \to \infty$

#2 $\tanh T = 1/\sqrt{2} \Rightarrow T = \tanh^{-1} 1/\sqrt{2}$

#3 $\tanh T = \sqrt{2} \Rightarrow$ Never