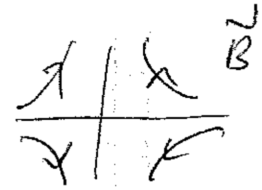
G.O. of Magsonic mode about X-point

$$\partial_t^2 \psi = V^2 \nabla^2 \psi, \quad V^2 = B^2/\rho\mu_0$$



$$\vec{B} = \hat{z} \times \vec{\nabla}(Hxy)$$

$$\vec{B} = [-x, y]H, \quad B^2 = r^2 H^2$$

WKB dr. $\Rightarrow \quad \omega^2 = \dfrac{k^2 B^2}{\mu_0 \rho} = \dfrac{k^2 H^2 r^2}{\mu_0 \rho}$

$$\frac{H^2}{\mu_0 \rho} = 1 \quad \Rightarrow \quad \boxed{\omega^2 = k^2 r^2} \quad (1)$$

2D $\Rightarrow$ $\vec{r} = \{x, y\}$, $\vec{k} = \{k_x, k_y\}$ 4 vars

$$\omega \omega_{\vec{k}} = \vec{k} r^2, \quad \omega \omega_{\vec{r}} = k^2 \vec{r}$$

$$\Rightarrow \boxed{\omega \dot{\vec{r}} = \vec{k} r^2, \quad \omega \dot{\vec{k}} = -\vec{r} k^2} \quad (2)$$

H eqns

(3) obvious Consts of motion $\boxed{\omega = kr}$ (3a) $= \text{const}$

$$\omega = \omega(k_x, k_y, x, y)$$

No obvious ignorables in Cartesian space

Try $\vec{D} = \vec{k} \cdot \vec{r}$

$$\dot{D} = \dot{\vec{k}} \cdot \vec{r} + \vec{k} \cdot \dot{\vec{r}}$$

$$= -\frac{k^2 r^2}{\omega} + \frac{k^2 r^2}{\omega} = 0$$

$$\therefore \boxed{\vec{k} \cdot \vec{r} = D = \text{const}} \quad (3b)$$

(4) Direct differentiation of $r(t)$

$$r^2 = \vec{r} \cdot \vec{r} \, ; \quad r\dot{r} = \dot{\vec{r}} \cdot \vec{r}$$

$$= \frac{\vec{r} \cdot \vec{k}\, r^2}{\omega} = \frac{D r^2}{\omega}$$

$$\Rightarrow \boxed{\dot{r} = \frac{D}{\omega} r}$$

let $\frac{D}{\omega} = \alpha$. $\Rightarrow |\alpha| = \frac{|D|}{|\omega|} = \frac{|\vec{k} \cdot \vec{r}|}{|kr|} < 1$

$$\therefore \boxed{\dot{r} = \alpha r} \quad (4)$$

any α, $|\alpha| < 1$

$$\Rightarrow r = r(0)\, e^{\alpha t}$$

$$r(0) = 1 \quad \Rightarrow \quad \boxed{r = e^{\alpha t}} \quad (5)$$

$$\alpha = 0 \Rightarrow r = 1 \rightarrow$$

$$\alpha > 0 \Rightarrow r \rightarrow e^{|\alpha| t}$$, increasing radius

$$\alpha < 0 \Rightarrow r \rightarrow e^{-|\alpha| t}$$, radius $\rightarrow 0$

(5) $\{x, k_x\}$ system

$$\omega \dot{x} = k_x r^2, \quad \omega \dot{k}_x = -x k^2$$

$r(t)$ is known, eliminate $\{k_y, y\}$

use $\omega^2 = k^2 r^2 \Rightarrow \cancel{\omega} \dot{k}_x = \dfrac{-x\omega^{\cancel{2}}}{r^2}$

$$\Rightarrow \omega \ddot{x} = \dot{k}_x r^2 + 2 k_x r \dot{r}$$

$$= -x\omega^{\cancel{2}} + 2\frac{r\dot{r}\,\omega \dot{x}}{r^2}$$

$$\Rightarrow \cancel{m}\ddot{x} = -\cancel{\phi} x + 2\frac{\dot{r}}{r}\cancel{\phi}\dot{x} \quad , \quad \frac{\dot{r}}{r} = \alpha$$

$$\Rightarrow \boxed{\ddot{x} - 2\alpha\dot{x} + x = 0} \quad (6)$$

(6) $\underline{\text{Try } x(t) \Rightarrow e^{\gamma t}}$

$$\Rightarrow \gamma^2 - 2\alpha\gamma + 1 = 0$$

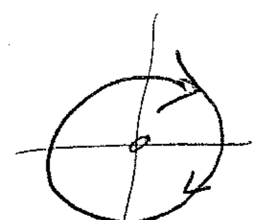
$$\Rightarrow \boxed{\gamma = \alpha \pm i\sqrt{1-\alpha^2}} \quad |\alpha| < 1$$

$$\Rightarrow \boxed{x(t) = e^{\alpha t} e^{\pm i\sqrt{1-\alpha^2}\, t}} \quad (7)$$

$$\mathrm{Re}(x) \Rightarrow \boxed{\mathrm{Re}\, x(t) = r(t)\cos\left[\sqrt{1-\alpha^2}\, t\right]}$$

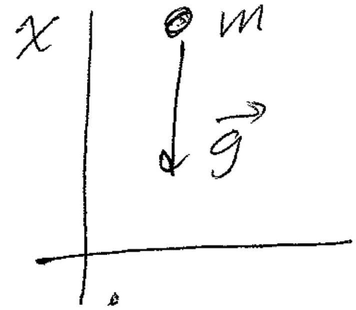
$$= r\cos\theta$$

$$\Rightarrow \theta = \left(\sqrt{1-\alpha^2}\right)t$$

$\alpha = 0 \Rightarrow \theta = t \Rightarrow r = 1 \Rightarrow$  $r=1$ fixed

$1 > \alpha > 0 \Rightarrow \theta = \sqrt{1-\alpha^2}\,t \Rightarrow$  spiral out

H–J Theory for g

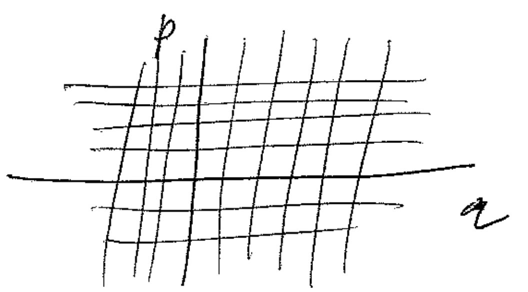


$$T = \frac{1}{2} m \dot{x}^2$$

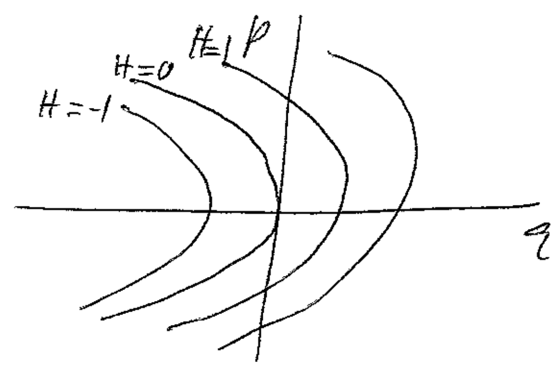
$$V = mgx \qquad p = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

$$H = \frac{1}{2}\frac{p^2}{m} + mgx$$

Let $m=1$, $g=1$ $\Rightarrow$ $\boxed{H = \frac{1}{2}p^2 + q}$



$\{p,q\}$ coords



H coord.

We want $\{p,q\} \to \{P,Q\} \ni P = \alpha = H$

Now $K = 0 \Rightarrow H + \frac{\partial S}{\partial t} = 0$

where $p = \frac{\partial S}{\partial q}$, $\beta = \frac{\partial S}{\partial \alpha}$, $S(q,\alpha,t)$

$$\Rightarrow \frac{1}{2}\left(\frac{\partial S}{\partial q}\right)^2 + q + \frac{\partial S}{\partial t} = 0$$

Let $S(q,\alpha,t) = W(q,\alpha) - \alpha t$

$$\Rightarrow \frac{1}{2}\left(\frac{\partial W}{\partial q}^2\right) = (\alpha - q)$$

$$\frac{\partial W}{\partial q} = \sqrt{2(\alpha - q)}$$. We pick the + sign, revisit if there is a problem

$$\Rightarrow \beta = \frac{\partial S}{\partial \alpha} = \frac{\partial W}{\partial \alpha} - t$$

$$\Rightarrow \beta + t = \frac{\partial}{\partial \alpha}\int^{q} dq' \sqrt{2(\alpha - q')}$$

$$= \int^{q} dq' \frac{\partial}{\partial \alpha}\sqrt{2(\alpha - q')}$$

$$= \int^{q} dq' (-)\frac{\partial}{\partial q'}\sqrt{2(\alpha - q')}$$

$$(\beta + t) = -\sqrt{2(\alpha - q)}$$

$$\Rightarrow (\beta + t)^2 = 2(\alpha - q)$$

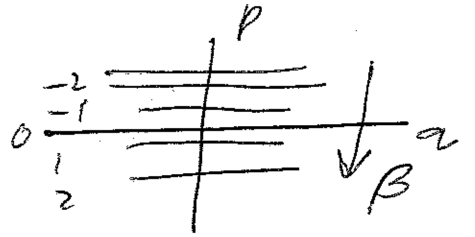
$$\Rightarrow \boxed{q = \alpha - \frac{1}{2}(\beta + t)^2} \Leftarrow q(\alpha, \beta, t)$$

$$p = \frac{\partial S}{\partial q} = \frac{\partial W}{\partial q} = \sqrt{2(\alpha - q)} = -\beta - t$$

$$\boxed{p = -\beta - t} \Leftarrow p(\alpha, \beta, t)$$

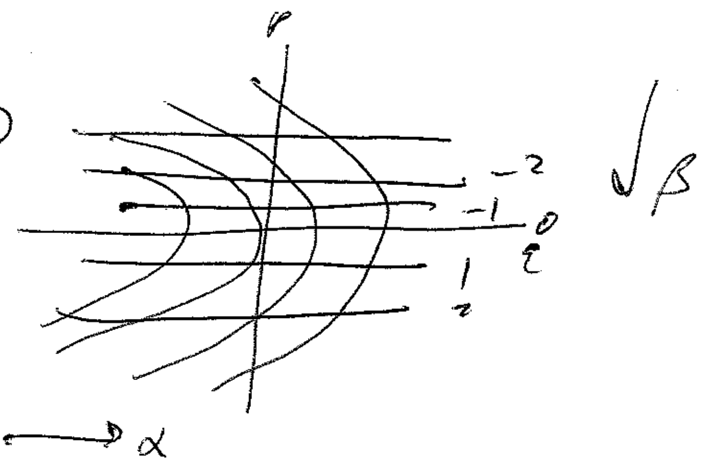
Now find $\alpha(p, q, t)$, $\beta(p, q, t)$

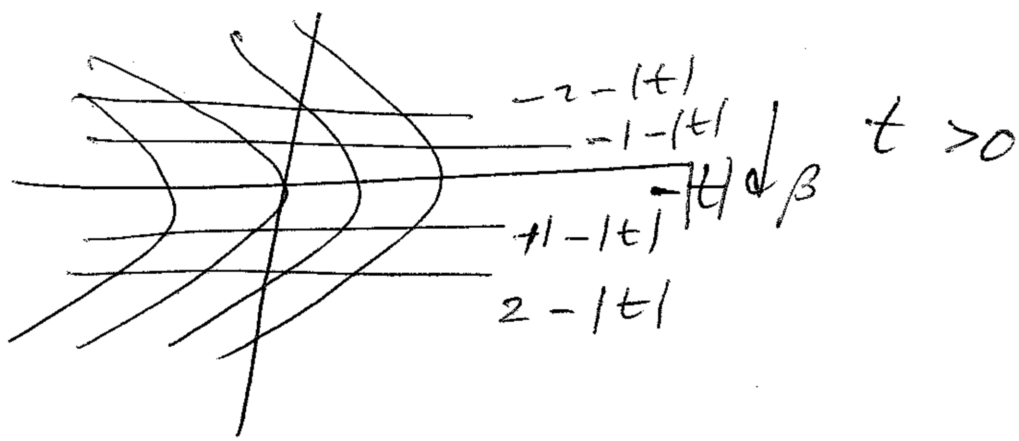
$\Rightarrow \quad \beta = -p - t$



$\alpha = q + \frac{1}{2}p^2$

$\Rightarrow$





β coordinates translate downward