

Problem 1 COVARIANCE

Consider the Galilean Transformation $x' = x - Vt$, $t' = t$, and the Lorentz Transformation $x' = \Gamma(x - Vt)$, $t' = \Gamma(t - Vx)$, $\Gamma^2 \equiv 1/(1 - V^2)$, where we have used $c=1$ units. These transformations connect two frames moving with V w.r.t. each other. Note that the motion of any mass m (rest mass) can be parameterized as $x(t)$, $x'(t)$, or $t'(t)$.

1. Show that Newton's equations $dx/dt = u$, $mdu/dt = F$ are covariant under GT. You may assume that F is a given constant. You may define F' in terms of F appropriately so as to accommodate covariance.
2. Show that Newton's equations are not covariant under LT, even if F' is appropriately defined in terms of F and V . (Find explicitly the equation in the unprimed frame if NE apply in the primed lab frame, or vice-versa.)
3. By contrast, show that Einstein's equations $dx/dt = u$, $md[\gamma u]/dt = F$, $\gamma^2 = 1/(1 - u^2)$ are covariant under LT.

[“Covariance” is defined as “the equations look the same”. i.e., both physicists agree on the type of all operations needed to describe motion under F .]