Phys601/F11/Problem Set 9 *due Nov 7 2011*

Subject to upgrade

9.1H

Find all the roots of the algebraic equation

$\varepsilon^2 x^6 - \varepsilon x^4 - x^3 + 8 = 0$

by perturbative methods assuming $\varepsilon << 1$. The roots may be complex. Obtain the roots correct to first non-vanishing order (i.e., get $x = x_0 + x_1$, at least, for both roots) and check for self-consistency (i.e., $|x_1| << |x_0|$).

9.2H

Solve the ODE below by perturbative methods if $\varepsilon << 1$.

$\varepsilon y'' + (1+x)y' + y = 0.$

Assume we are only interested in solutions valid in the range $x \geq 0$. Obtain both solutions correct to first order (i.e., $y = y_0 + y_1$). Check each for self-consistency of solution. Make a sketch of both solutions in the domain $x = [0,2]$, assuming $\varepsilon = 1/10$. [You may find an unexpected form for y_1 for the WKB solution. Try y_2. If that's also similar, try to explain why.]

9.3H

Electromagnetic waves propagating in a medium with dielectric $\varepsilon(x)$ satisfy the wave equation

$$\varepsilon(x)\, \partial^2\psi/\partial t^2 = c^2 \partial^2\psi/\partial x^2, \qquad \psi = \psi(x,t).$$

Suppose $\varepsilon(x) = (1 + x^2/2L^2)^2$. The medium is disturbed at x=0 by an antenna with frequency ω. If $\omega L/c >> 1$, find the WKB solution for $x > 0$ for waves propagating to the right, correct to 1st order. By demanding that $|k_1| << |k_0|$, obtain the condition that the WKB solution is a good approximation. At what x/L do you find the worst violation? Make a sketch of the real part of ψ, to first order, over a scale of length a few L's.

Find x(t), the location of the wavepacket moving with the local group velocity if x(0) = 0. From the constancy of ω, find k(t).