Phys601/F11/Problem Set 8 **Due Oct 31, 2011**

(Upgrades in red)

8.1H

A particle has the Hamiltonian $H = p^2/2 - q^2/2$.

1. Sketch the contours of constant H for all values of H. Sketch contours for the function Q' = pq. Show that contour lines of H and Q' are orthogonal to each other by computing the dot product of the phase space gradients of H and Q'.

2. Make a canonical transformation to coordinates P and Q where P = H. Find Q, then p(P,Q) and q(P,Q) and their inverses (be careful with the sign of P and any square roots). Make a sketch of the contour lines of Q. This is obviously not orthogonal.

8.2H

Consider a mass m moving vertically in the Earths gravitational field, **g**. Write down the Hamiltonian and Hamilton's equations. Sketch the contours of constant H. Make a canonical transformation to coordinates P and Q where P = H. Find Q, then p(P,Q) and q(P,Q) and their inverses. Make a sketch of the contour lines of Q. What are Hamilton's equations in P(t) and Q(t)? Compare with the H equations derived earlier.

8.3H

Consider a particle mass m moving in 1D in a potential field V(x) = V'|x|. V'=constant. Let x=q. You may normalize, ie, set things to 1.

1. Write down the Hamiltonian. Sketch contours of constant H.

2. Make a canonical transformation to action-angle variables P and Q, using F_2 as was done in class. Find P(H). Find Q(t) and identify the period for one complete bounce as a function of H. Compare with the harmonic oscillator.

3. Explicitly find Q(p,q). You may restrict to the quadrant p>0 and q>0. Set the line q=0 as the contour Q=0. Show that on the line p=0, q >0, the value of Q is a constant, ie, the line p=0 coincides with a constant Q contour. This ensures that Q is periodic, as we want. What is the value of Q for this line? (call this Q_0) (Depending on whether you used a 2π in your definition of P, this value could be $\pi/2$.) Now make a sketch of a contour line $0 < Q < Q_0$. Describe qualitatively what this line looks like when Q->0 and when Q -> Q_0 (not equal to 0 or Q_0)?

4. Are lines of Q and P ever orthogonal to each other?

5. Make a canonical transformation from {p,q} to action-angle variables {P,Q} using F_4 as the generating function. Check with your answers from 2 and 3.

8.4H

(This problem is done in GPS. It will be instructive to do it yourself.)

Suppose we do a CT from {p,q} to {P,Q}. Hamilton's equations in the new system are covariant, ie, $dQ/dt = \partial H/\partial P$, $dP/dt = - \partial H/\partial Q$.

1. Start with the first of these equations. Use Q(p,q) to obtain an expression for dQ/dt, using partials with respect to q and p. Next, start with H[q(P,Q), p(P,Q)] and compute the RHS of the first equation, ie, $\partial H/\partial P$, again using partials with respect to q and p. Now compare the LHS and the RHS. If these are to agree, show that the relations (9.48a) from GPS must be satisfied for any CT.

2. Repeat the above procedure now using the 2nd Hamilton equation for dP/dt, and so obtain GPS (9.48b).