Phys601/F11/Problem Set 7 **Due: 10/24/11**

(subject to upgrade)

7.1H

A particle of mass m moves in an x-dependent potential $V(x) = |x| V'(t)$, assume $V'(t)>0$.

a. obtain the Hamiltonian for the system

b. write down the equations of motion

c. for the special case V'=constant, draw level curves of the Hamiltonian and the trajectory of the particle in phase space.

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7.2H

Consider a 3-dimensional magnetic field $\mathbf{B} = [Bx(x,y,z), By(x,y,z), Bz]$ in the special case where $B_z = B_0$ = constant. Write this magnetic field as $\mathbf{B} = B_0\hat{\mathbf{z}} + \nabla \times (\hat{\mathbf{z}} A_z)$, where $\hat{\mathbf{z}}$ is the unit vector in the z-direction . The "trajectory" of a field line, {x(z),y(z)}, can be generated from the condition $d\mathbf{r} \times \mathbf{B} = 0$. Show that x(z) and y(z) are conjugate variables satisfying a Hamiltonian system of equations.

7.3H

A mass m rolls up a hill whose relative height $y = - x^2/(2a)$, where x is the horizontal coordinate. The gravitational acceleration is g.

1. Find the Hamiltonian for this system.
2. Normalize the Hamiltonian so that energy, length, and time are normalized to mga, a, and $(a/g)^{1/2}$.
3. Write down an approximate Hamiltonian if we are only interested in motions for which $|x| << a$ (ie, $|x| << 1$). Lowest order only.
4. Sketch the contours of H(p,x) from 3 in normalized phase space.
5. Consider the 3 initial conditions [x(0), v(0)] = [1,-1], [1,-21/2], [21/2, -1], where v is the velocity. Show in phase space the subsequent trajectories of these 3 cases (for the approximate H from above).
6. Find x(t) for all 3 cases. Find the time it takes for the mass to get to x=0 (if it does).