<u>Phys601/F11/Problem Set 6</u> Due 10/17/11

(upgraded)

<u>GPS:</u> Do Ch8 #1, #27

<u>6.1H</u> Van der Pol Oscillator

Consider the Van der Pol Oscillator for x(t)

$$d^2x/dt^2 + x = -2\nu\,(dx/dt)\,(1-x^2).$$

Let x ~ 1 ("of order unity"), i.e., make no assumptions on whether x is small or large. This means that the sign of the anti-friction term on the RHS could change.

a. Suppose $\nu \ll 1$. Solve for x perturbatively by expanding in a series $x = x_0 + x_1 + x_2$ ….. assuming that each term is smaller than the previous term in the series. Show that the regular perturbation series exhibits secular behaviour.

b. Solve by assuming a multiple time scale solution, i.e., assuming that $x = x(t, \tau)$, where τ is over a longer time scale, in particular $d/dt = \partial/\partial t + \partial/\partial\tau$, where $|\partial/\partial\tau| \ll |\partial/\partial t|$. Obtain solutions to lowest significant order, i.e., make sure you get to the interesting stuff. Make a sketch of x(t) for the two cases $0 < x(0) < 1$ and $1 < x(0)$.

You will have to solve an equation using the "energy method". Look up the integrals needed.

<u>6.2H</u> Multiscale Bertrand's calculation (optional)

In class, we started with the equation $d^2u/d\phi^2 + u = u^x$, where $u = 1/r(\phi)$ and r is the satellite orbit around a mass. x = 0 corresponds to the Kepler problem and x=-3 corresponds to the 3D harmonic oscillator. x < 1 for stability. There is a circular orbit solution u = 1.

Consider small perturbations about u = 1. Expand the perturbations in a series $u = 1 + u_1 + u_2 + u_3$ …. In class, we showed that regular perturbation theory leads to secular behaviour if x is not 0 or -3. Attempt to remove the secularity by using a multiscale method. You might find you don't need the multiscale until you get to high enough order, but allow for the possibility. There may be equations you cannot solve analytically but you may only need the period that you could get from an energy method. I am not sure the multiscale method works. You may stick to x=-8 if that helps with the algebra. Signs are important – things should not blow up.

Alternatively, let us know if you find any references in the literature on multiscale methods in Bertrand's theorem.