

**Phys601/F11/Problem Set 4**  
(to be upgraded)

**Due 10/03/11**

**From Goldstein et al**

Ch2: 3,5,12,18,24

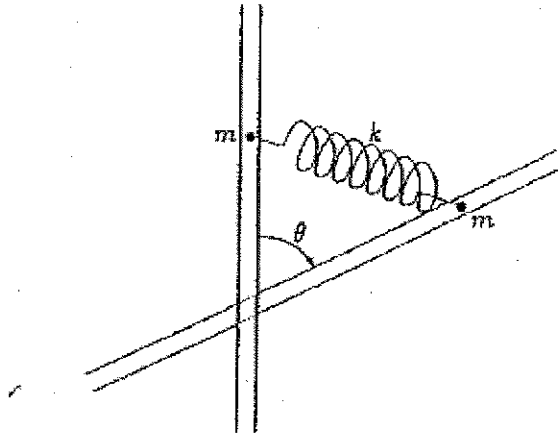
**4.1H**

Consider a charged particle  $q$  of mass  $m$  in a given electromagnetic field  $\mathbf{E} = E\hat{y}$ ,  $\mathbf{B} = B\hat{z}$ , where  $E$  and  $B$  are constants. The particle starts out at  $x=0$  and  $y=0$  with zero velocity. Assume the EM force is  $q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$ .

1.  $\mathbf{E}$  and  $\mathbf{B}$  in general can be written in terms of  $\mathbf{A}$  and  $\phi$  according to  $\mathbf{B} = \nabla\times\mathbf{A}$  and  $\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t$ . Find  $\mathbf{A}$  and  $\phi$  for the given  $\mathbf{E}$  and  $\mathbf{B}$ . You may assume that only  $A_y$  is nonzero.
2. Write down the Lagrangian
3. Find the Euler-Lagrange equations for  $x(t)$  and  $y(t)$ .
4. Find  $x(t)$  and  $y(t)$ . Your answer should be a cycloid.

4.10 (adapted from Qualifier Problem, ~~attached~~)

The figure shows two tracks making an angle  $\theta$  with each other, in which particles of equal mass  $m$  are constrained to move without friction. A massless spring of spring constant  $k$  and zero unstretched length connects the two particles. (The tracks are of infinite length, and the particles pass over each other if they reach the intersection point of the tracks together; the length of the spring is zero at that moment of crossing):



- Choose suitable generalized coordinates that can describe the motion of this system. (Specify the coordinates by a diagram or in words). Write the Lagrangian for the system. [5 points]
- Derive the equations of motion. [5 points]

Add these to the problem:

- Check your answer by making sure that the limits  $\theta = 0$  and  $\theta = \pi/2$  make sense.
- A set of linear, coupled ODEs with variables  $q_j(t)$ ,  $j=1, \dots, n$ , can always be solved by assuming solutions of the form  $q_j = q_{j0} \exp[-i\omega t]$  provided the coefficients of each of the terms are constant. Use this to find the general solution of your equations of motion.