Phys601/F11/Problem Set 3 **Due 09/26/07**

(to be upgraded)

From Goldstein et al

Do Ch 1 #8, #9, #10 (use #9), #20 (possible typo in #20: should be V^2)

3.1H

Prove the following identities. **A** is a constant vector, **r** is a vector with Cartesian components x_k, the components of $\partial/\partial\mathbf{r}$ are $\partial/\partial x_k$, and $r = |\mathbf{r}| = (\mathbf{r.r})^{1/2}$. Note that $\partial\mathbf{r}/\partial\mathbf{r} = \mathbf{1}$, where **1** is the unit tensor with Cartesian components $\mathbf{1}_{jk} = \delta_{jk}$.

a. $\partial(\mathbf{r.A})/\partial\mathbf{r} = \mathbf{A}$
b. $\partial(r^2/2)/\partial\mathbf{r} = \mathbf{r}$
c. $(d/dt)(r^2/2) = \mathbf{r.v}$ if $\mathbf{v} = d\mathbf{r}/dt$
d. $(d/dt)f[\mathbf{r}(t),t] = \mathbf{v.}\partial f/\partial\mathbf{r} + \partial f/\partial t$ if $\mathbf{v} = d\mathbf{r}/dt$

3.2H

Suppose a force, **F**, acting on a mass m has the form $\mathbf{F} = Q\,\mathbf{v}.[\mathbf{r\,D} - \mathbf{D\,r}]$, where the order of the vectors in the dyadic is important, Q=constant, and **D** is a constant vector. Here, **r**(t) is the particle position vector, and **v**(t) = d**r**/dt is the velocity.

We contend that if this is fundamental force of physics, the Newtonian equation of motion must be rewritable as[1]

$$(d/dt)(\partial L/\partial \dot{q}_k) = (\partial L/\partial q_k),$$

where L = T – U, T is the kinetic energy, and $U = U(\dot{q}_k, q_k, t)$. Here $q_k(t)$ are generalized coordinates and there are no constraints assumed in the formulation of **F**. (Since there are no constraints, one may let $q_k \rightarrow \mathbf{r}$.)

Show that **F** can be cast into Lagrangian formulation by finding a U(**r**,**v**). There is a systematic way to do this but it can be done by trial and error.

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[1] Not all **v**-dependent forces can do this – for example, the friction force, $-\mu\mathbf{v}$, cannot.