Phys601/F11/Problem Set 2 **Due 09/19/11**

(upgraded)

2.1H Kepler Problem

A particle of mass m moves in the 1/r gravitational potential of a fixed massive point object of mass M. Let k=GMm and **p** = m**v**. Let **L** = **r** x **p** be the angular momentum with **r**(t) being the position vector from the center of the mass M. Use the constancy of angular momentum L and the energy E to formulate first order differential equations in t for r(t) and ϕ(t). Find from these a differential equation in ϕ for the orbit r(ϕ). By making the substitution u = 1/r, solve the resulting equation (look up the integral) to find an expression for r(ϕ). Compare with Eq (3.55) in the text. For what values of E would you get a circle, an ellipse, a parabola, and a hyperbola.

2.2H Laplace-Runge-Lenz vector

A particle of mass m moves in the gravitational field of a fixed massive point object of mass M. Let k=GMm and **p** = m**v**. Let **L** = **r** x **p** be the angular momentum with **r**(t) being the position vector from the center of the mass M.

(a) Show by direct differentiation that the vector **A** = **p** x **L** – km**r**/r is a constant of the motion. The relation **v** = (dr/dt)$\hat{\mathbf{r}}$ + r(dϕ/dt)$\hat{\boldsymbol{\phi}}$, where r and ϕ are polar coordinates in the plane of the motion and $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\phi}}$ are unit vectors, may be useful. Confirm that this constancy would not work for any other power of r except the 1/r potential.

(b) Now derive the constancy of **A** as follows. Starting from the equation of motion, cross both sides by **L** and then manipulate the resulting equation, using the equations of motion, to derive the equation d**A**/dt = 0.

(c) Show that not all 3 components of **A** are new constants of motion. In particular, show that the constant corresponding to the component parallel to **L** is trivial. Next, show that the magnitude of **A**, A^2, is a combination of L and energy E. Thus, only the direction of **A** in the plane of the orbit is a new constant of motion.

(d) Consider the combination **r.A**, using the direction to define ϕ as **r.A** = r cosϕ. Show that this yields an expression which connects r(t) and ϕ(t) to yield the spatial orbit r(ϕ). Check your answer with the textbook and **2.1H** above. For what values of A/mk $\geq$ 0 would you get a circle, an ellipse, a parabola, and a hyperbola.

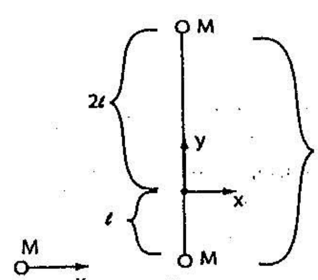
2.3H EM energy density

In Goldstein, conservation of energy for a many particle system is derived assuming that the interparticle force is derivable from a potential. We want to extend this calculation to the electromagnetic force, $F/q = \vec{E} + \vec{v} \times \vec{B}$. Proceed to do this as in the text, but average the particles over a small volume to define charge density, ρ, and current density, $\vec{j}$, according to $q \to d^3r\,\rho$ and $q\vec{v} \to d^3r\,\vec{j}$. Thus, convert sums to integrals. Now manipulate the RHS using the Maxwell Eqns to substitute for ρ and $\vec{j}$. [This was done in class for momentum. Or you can look at Jackson or Griffiths.] Upon using vector identities for the RHS and assuming that $\vec{E}$ and $\vec{B}$ die fast enough at infinity, show that the sum of $T + \int d^3r\,(E^2 + B^2)/2 = const.$, where T is the total kinetic energy of the particles. Thus, identify the expression in the square brackets as the EM energy density.

2.1Q

Two identical blocks of mass M (and negligible dimensions) are connected by a massless rigid rod of length 3ℓ. A bullet, also of mass M, is fired into one of the blocks with velocity v, normal to the line joining the blocks. Before the collision, and in the laboratory frame, the blocks are at rest. The bullet enters and becomes lodged in the block so that they move together after the collision. Ignore any effect of gravity.

Consider a time before collision. A convenient laboratory coordinate system to use for this problem has the y-axis along the line initially joining the two blocks, and the origin of coordinates a distance ℓ from one block and 2ℓ from the other. The bullet moves initially in the x, y plane. See the figure below that shows the system before collision.



a. Assuming that collision occurs at time $t = 0$, find explicitly (in terms of (e_x, e_y, and e_z) the position of the center of mass of the "bullet+blocks" system as a function of time and in the laboratory frame before and after collision. [5 points]

b. Consider the center of mass frame. In this frame find explicitly the total angular momentum of the system, relative to the center of mass, before collision and after collision. [5 points]

c. Describe in a qualitative way the motion in the center of mass frame after collision, and justify your answer. [5 points]

d. Find explicitly, as a function of time and in the laboratory frame, the positions of the two blocks after collision. [5 points]