Phys601/F11/Problem Set 1 Due 09/12/11

Problem 1 COVARIANCE (posted separately, to be renamed to 1.1H)

1.2H

A particle mass m = 1 moves in a force field $\mathbf{F} = \mathbf{r}/x$, where (x,y) are Cartesian coordinates and x(t) is never zero. Initial conditions are specified at t = 1. These are x(1) = 1/2, y(1) = 0, z(1) = 0, (dx/dt)(1) = 1, (dy/dt)(1) = 3, (dz/dt)(1) = 0.

(a) Find {x(t), y(t), z(t)} by direct solution of the 2nd order ODE's.

(b) Find at least one constant of the motion and use this to solve for **r**(t). (You may still need to solve one 2nd order ODE directly.)

1.3H

A particle of mass m = 1 is moving in a force field $\mathbf{F(x)} = -\nabla U$, where U = xy. x and y are Cartesian coordinates. At t=0, x(0)=1, y(0)=1, z(0) = 0, and **v**(0) = 0.

(a) Find x(t), y(t), z(t) for all subsequent t by direct solution.

(b) Identify a constant of the motion.

(c) Show, by direct differentiation, that the combination $M = (dx/dt)*(dy/dt) + (x^2 + y^2)/2$, is also a constant of the motion.

(d) Introduce the variables p(t) = x(t) + y(t) and q(t) = x(t) – y(t). Express the constants of the motion in p and q and find p(t) and q(t), and so x(t) and y(t), using the constants of the motion.

1.4H

A charged particle moving in a magnetic field is described by the equations $d\mathbf{v}/dt = \mathbf{v} \times \hat{\mathbf{z}}$, $d\mathbf{r}/dt = \mathbf{v}$, where $\mathbf{r}$ and $\mathbf{v}$ have their usual meanings, and we have assumed that the magnetic field is given by $\mathbf{B} = \hat{\mathbf{z}}$. $\hat{\mathbf{z}}$ is the unit vector along the z –axis, and some constants have been set to unity.

(a) suppose $v_z = 0$ at $t = 0$. Prove that the subsequent motion is confined to a plane orthogonal to $\mathbf{z}$. Let this be the x-y plane.

(b) Now use polar coordinates, r and φ, to obtain the set of coupled differential equations satisfied by r(t) and φ(t).

(c) Show, by inspection, that your equations admit a solution such that r(t) = C and $d\varphi(t)/dt = D$, where C, D are constants. What is the value of D?