<u>Phys601/F11/Midterm</u>

Take-Home Due in class 11/21/11

1. Work independently but can consult any text, etc

2. Return only this exam booklet (8 pages), with boxes filled out. All important steps should be clearly shown, succinctly and neatly. If extra space is needed (it should not be), use the back sides. Do not attach other pages unless you absolutely have to.

3. Send email to me for any clarifications/typos.

4. Parts 1, 2, 3, 7 can be done independently. Possibly some others.

5. 10 parts, O(10) points per part.

Monopole

A particle of mass m and charge q moves in the field of a magnetic monopole. This field is given by $\mathbf{B} = b\mathbf{r}/r^3$, where b is a constant. Assume that the mass m never gets to $\mathbf{r} = 0$. Apart from the Lorentz force, there are no other forces on the mass m.

For the first 6 parts, do not use the Lagrangian method. Simply use Newton's Equations in the original form.

1. Prove directly from Newton's Equations that the kinetic energy, $T = (1/2)mv^2$, is a constant of the motion.

2. Prove by direct differentiation that the vector $\mathbf{D} = \mathbf{L} - qb\mathbf{r}/r$ is a constant of the motion. Here $\mathbf{L} = m\mathbf{r} \times \mathbf{v}$ is the angular momentum.

3. Suppose we place the z-axis of a spherical coordinate system to point in the direction of the constant vector **D.** Suppose at t=0 the particle is kicked off with $\mathbf{v}(0) = -\hat{\phi} v_0$, $r(0)=r_0$, and $\theta(0)= \theta_0$, where v_0 is related to the initial position according to $r_0 v_0=(qb/m)\tan\theta_0$. (r, θ, ϕ) are spherical coordinates. Identify all constants of the motion using spherical coordinates. In particular, count how many you have, point to any redundancies, and express these constants in terms of the initial conditions provided. *[Note:* $\mathbf{v}(0)$ *and* $\hat{\phi}(0)$ *are negative, though* v_0 *is defined positive. The signs are important.]*

Constant #1:

Constant #2:

Constant #3:

Constant #4:

4. Find $\theta(t)$. Based on this, what can you say qualitatively about the motion of the particle? Illustrate by a sketch.

$\theta(t) =$

5. Next, find an equivalent 1-D equation for r(t), ie, this equation should include only $(dr/dt)^2$ terms and an effective potential which is a function of r. What is the effective potential $V_{eff}(r)$? Qualitatively describe the motion of the particle and find $|\mathbf{v}|$ as $r \to \infty$. [Note: r_0, v_0, and θ_0 are related. It is convenient to work with the first two.]

Equivalent 1-D Eq and V_{eff}

6. Find the solution for r(t). Compare with your conclusion in 5 as $t \to \infty$.

r(t)

For the remaining parts, Lagrangian methods are to be used.

7. Find a vector potential for the magnetic field, valid for $\mathbf{r} \neq 0$, by assuming that $\mathbf{A}$ is always only in the $\hat{\phi}$ direction. [Why are we writing down a vector potential for a monopole field which has a divergence somewhere?]

A =
Why?

8. Find the Lagrangian and all obvious constants of the motion that can be found via the Lagrangian. Express the constants in terms if the initial conditions provided. To what extent do these agree with those in part 3?

Lagrangian =

Constant #1:

Constant #2:

Constant #3:

9. From the Lagrangian, write down the E-L equation for the θ variable. Use the initial conditions as provided earlier [$\mathbf{v}(0) = -\hat{\phi} v_0$, $r(0)=r_0$, and $\theta(0)= \theta_0$, where v_0 is related to the initial position according to $r_0 v_0=(qb/m)\tan\theta_0$]. Eliminate $d\phi(t)/dt$ by using a constant of the motion, so that the resulting equation includes only {r, θ} and their derivatives. With all this, show by direct plug-in that $\theta(t)$ as obtained in Part 4 solves your θ equation.

θ equation: $\theta(t) =$

10. Consider small perturbations about the $r(t)$ and the $\theta(t)$ obtained previously, ie, $r(t) \to r(t) + \tilde{r}(t)$, etc. Use the $\theta(t)$ equation from part 9 as the starting point. Is the $\tilde{\theta}(t)$ solution stable? Find the frequency (or growth rate) for small oscillations. Hint: even though $r = r(t)$ in the "equilibrium", it can be "absorbed" into a derivative.

Stable? Frequency=?