Phys601/F11/Final

Take-Home ~ 72 hours

1. Work independently but can consult any text, etc

2. Return only this exam booklet, with boxes filled out. Do your scratch work elsewhere but then collect all important steps and summarize these in the booklet, succinctly and neatly. If extra space is needed (it should not be), use the back sides. Do not attach other pages unless you absolutely have to.

3. Fill in the boxes as instructed. The boxes may ask for intermediate steps.

4. Send email to me for any clarifications/typos.

Problem 1 **Torque** **(20 points)**

(Goldstein Ch5 Problem 18)

This problem is to be done in 3 ways, 1.1 to 1.3 below

First, on this page, draw some pictures, to show **x^, y^, z^, e_1^, e_2^, e_3^, ω**, θ, etc.

1.1 In this part, find the torque using body-based dynamical equations.

Show the important components of the Inertia tensor. Specify if body-based, space-based, etc.

Show the torque, in body based system.

1.2 Now, calculate in the space based system and evaluate **N** from this.

Show the important components of the Inertia tensor. Specify if body-based, space-based, etc.

Show the torque and compare with the body based system.

1.3 Use free-body diagrams to note that there must be an additional force on each mass to make the system rotate as specified; find this force and therefore find **N**.

Show a sketch depicting any extra forces and then specify the torque

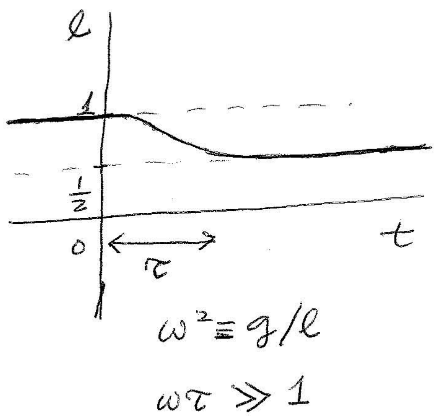
Problem 2 **Adiabatic Invariance** **(30 points)**

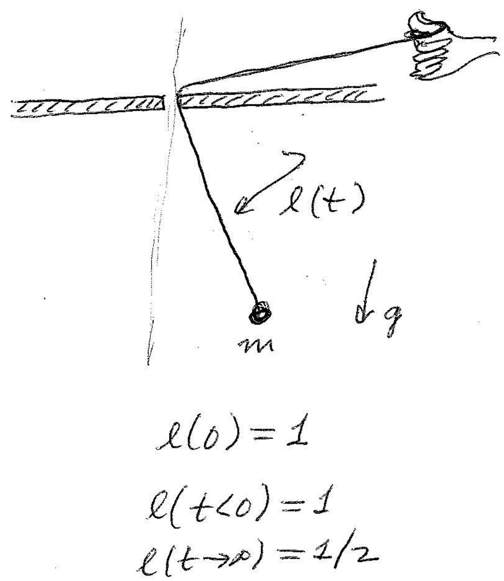
A pendulum swings on a string from a hole in the ceiling. The length of the string is one normalized unit at t=0. After t=0, the string is pulled up slowly so that its final length is 1/2 units. Thus, assuming g=1, the frequency of oscillation changes from $\omega = 1$ to $\omega = 2^{1/2}$. By "slowly", we mean many oscillations occur over the time it takes for the string to be pulled up, ie, $\omega^2 >> (d\omega/dt)$ (see the pics below).

Thus, assume that the equation satisfied by the small angle $\theta(t)$ is given by

$$d^2\theta/dt^2 \; + \; \omega^2(t)\,\theta \; = \; 0, \qquad \omega^2(t) \; = \; g/l(t).$$

Assume $\omega(t) > 0$ is a given function. Assume m = 1.





2.1 Use the WKB eikonal method to find an approximate solution for $\theta(t)$, in terms of given $\omega(t)$, *correct to first order*. The lowest order solution is not acceptable. Your solution should satisfy the conditions $\theta(0)=0$ and $(d\theta/dt)(0)=1$. Assume that $(d\omega/dt)(0)=0$.

WKB solution to first order including initial conditions

2.2 Write down the Hamiltonian function for this system, $H(p,\theta,t)$. Using your solution for $\theta(t)$, calculate explicitly, *to lowest non-vanishing order only*, the energy of the mass, $h(t)$. The latter function is the "h" function as defined by GPS.

$H(p,\theta,t) =$

$h(t) =$

Now, make a sketch of contours of constant H for the two cases $t = 0$ and $t >> \tau$. Calculate the value of $h(t)$ for these two cases; then mark the position of the particle on your H-contours, ie, identify which H surface it begins on and where it ends up. Your two H-contours should be approximately to scale, showing the distortion as well as H-values and where "h" is.

Two sketches

2.3 From your WKB solution, calculate *to lowest non-vanishing order* the action, A(t) = $\int p d\theta$, for this system at any time, where the integral is over one full period. [The integral is doable even if ω(t) is not known explicitly.] Show that, to the order of your calculation, the action is a constant of the motion. What is the interpretation of $\int p d\theta$ and its constancy in phase space? Complete the sentence below and show your 2 sketches again, this time with some shading.

What is A(t)?

Complete the sentence: "As time cvolves, the H surfaces slowly distort but the particle stays on those H surfaces such that ____."

<u>Problem 3</u> **Coriolis** **(20 points)**
(there are 2 pages for this)

A particle is thrown up vertically near Earth in the Northern Hemisphere with initial speed v_0, reaches a maximum height, and falls back to ground. Find the ratio of the deflection (from non-inertial effects) the particle experiences compared to the deflection for when it is dropped from rest from the same maximum height.

Note: *Your calculation should be perturbative, in the smallness of the rotation frequency of the Earth.* A solution correct *only to the first order* is needed. This is also the lowest non-vanishing order which gives deflection; obviously there is no deflection to lowest order. Identify the smallness dimensionless parameter. Your ratio should include a sign, depending on whether the deflections are or are not in the same direction.

What is the smallness parameter?

Ratio of Coriolis deflections =

<u>Problem 4</u> **Hamilton** **(20 points)**

Do Goldstein Ch10 #6, but only as outlined below:

4.1 Write down the Hamiltonian and Hamilton's equations in polar coordinates

Hamiltonian H =

Hamiltons equations

4.2 Use 1. to solve for $r(t)$ and $\theta(t)$ for the initial conditions $r(0)=1$, $dr/dt(0)=0$, $p_\theta(0)=0$, $\theta(0)=0$, where p_θ is the canonical momentum.

r(t), etc

4.3 Describe the geometry of the orbit in simple terms. Draw a picture.

picture

<u>Problem 5</u> **Hamilton-Jacobi** **(20 points)**

Do Goldstein problem 10.5, as specified below.

(a) Show that S is a solution

(b) Find $q(\alpha,\beta,t)$ and $p(\alpha,\beta,t)$ explicitly, where β is another constant

p=   q=

(c) Why is this the correct solution?

Why?