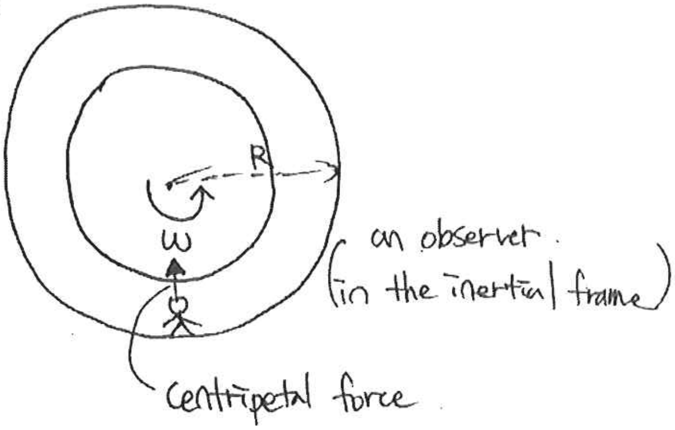
Solns PS9

Phys 410/516

9.2

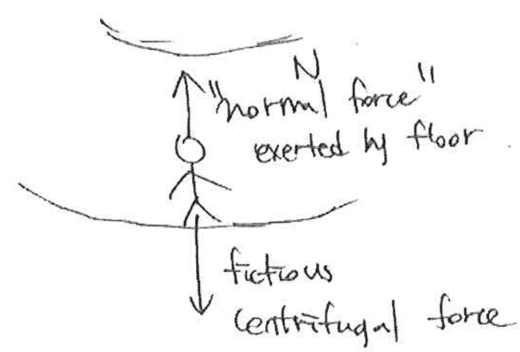
(a)



$$F = m\frac{v^2}{R} = mR\omega^2$$

$$\therefore\ a = R\omega^2$$

(b) <in the rest frame>



$$ma = N - m\omega^2 R$$
$$= 0$$

(the astronaut is at rest)

$$m\omega^2 R = mg$$

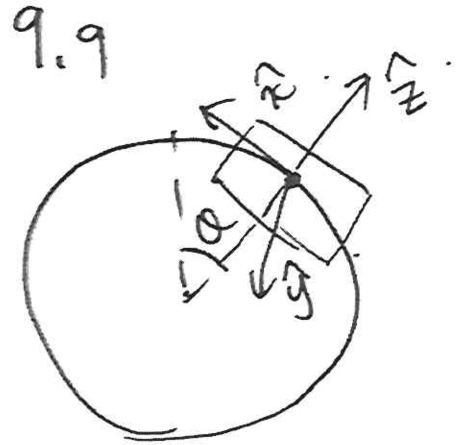
$$\omega^2 = \frac{g}{R}, \quad \omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{40}} = \frac{1}{2}s^{-1} = \frac{1}{2}\ (\text{rad/s})$$

(b)

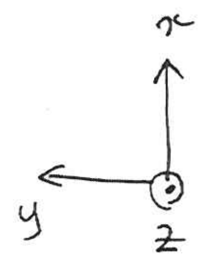
$$g_1 = 40\cdot\left(\frac{1}{2}\right)^2 m/s^2 = 10\ m/s^2$$
$$g_2 = 38\cdot\left(\frac{1}{2}\right)^2 m/s^2 = 9.5\ m/s^2$$

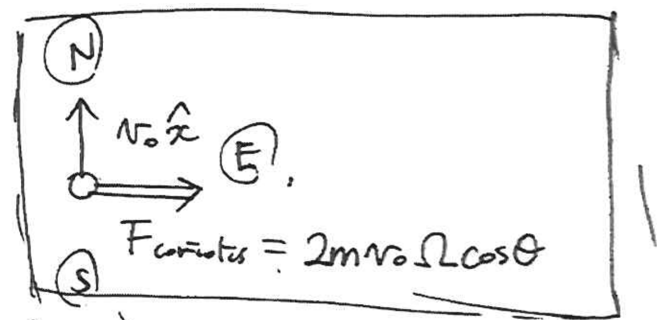
$$\text{diff}\ (\%) = \frac{10-9.5}{10} \times 100\% = 5\%$$

9.9



$\vec{\omega} = \Omega \cos\theta \hat{z}$





$$\vec{F} = -2m\, \vec{\omega} \times \vec{v}$$

$$= -2m\, \Omega \cos\theta\, \hat{z} \times v_0 \hat{x}$$

$$= -2m v_0 \Omega \cos\theta\, \hat{y}.$$

$v_0 = 1000$ m/s , $\theta = 40°$,

$$\Rightarrow F_{coriolis} = 2 \times 1000 \text{ m/s} \times \frac{2\pi}{24 \times 3600 \text{s}} \times \underbrace{\cos 40°}_{0.7660} \text{ m}.$$

$$= 0.111 \text{ m}.$$

(m/s^2).

9.12

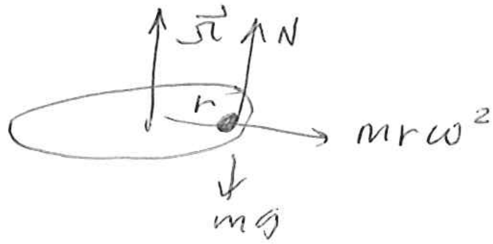
(a) statics $\Rightarrow \ddot{\vec{r}} = 0, \ \dot{\vec{r}} = 0$

$$\Rightarrow \quad 0 = \cancel{2\vec{v} \times \vec{\Omega}}^{\,0} + (\vec{\Omega} \times \vec{r}) \times \vec{\Omega}$$

$+ \ \vec{N}$ forces, etc

$$\Rightarrow \quad 0 = (\vec{\Omega} \times \vec{r}) \times \vec{\Omega} + \vec{N}$$

(b)



Friction $= \mu N$

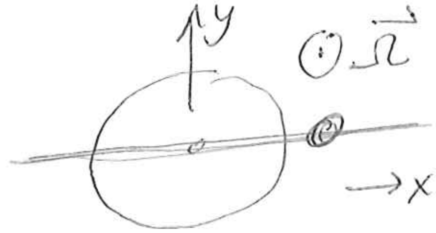
$m r \omega^2 = \mu N$ equilibrium

$m g = N$

$\Rightarrow r\omega^2 = \mu g$

if $r > \mu g / \omega^2 \Rightarrow$ loss of static equil

9.16



$\odot\vec{\Omega}$ $\quad \vec{g}\otimes$

$\Omega = \text{const}$

Bead on rod $\Rightarrow$ $\ddot{\vec{r}} = 2\dot{\vec{r}}\times\vec{\Omega} + (\vec{\Omega}\times\vec{r})\times\vec{\Omega}$

$\Rightarrow \ddot{\vec{r}} = 2\dot{\vec{r}}\times\hat{z}\,\Omega + \vec{r}\,\Omega^2$

$\Rightarrow \ddot{x} = 2\dot{y}\Omega + x\Omega^2$

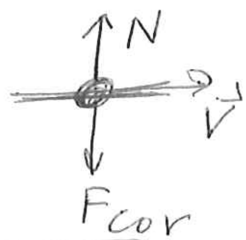
$\ddot{y} = -2\dot{x}\Omega + y\Omega^2 + N$

But $y = 0 = \dot{y}$ (bead on rod)

$$\Rightarrow \boxed{\begin{array}{l} \ddot{x} = x\Omega^2 \\ 2\dot{x}\Omega = N \end{array}}$$

N = normal force (on bead)

$\dot{x} > 0 \Rightarrow N > 0$

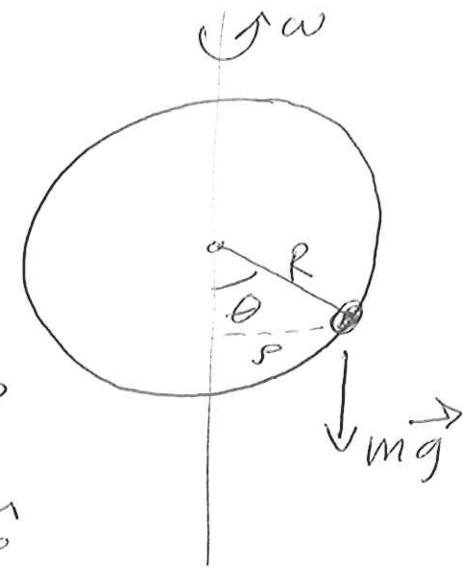


$\cancel{x = x_0 e^{\Omega t}}$

$x(0) = x_0 \neq 0$

$\dot{x}(0) = 0$ $\quad \Rightarrow \boxed{x = x_0 \cosh(\Omega t)}$

9.17



In rotating frame

① $\ddot{\vec r} = \vec g + 2\vec v \times \vec\omega + \omega^2 \vec\rho$

where $\vec\rho = R\sin\theta\,\hat\rho$

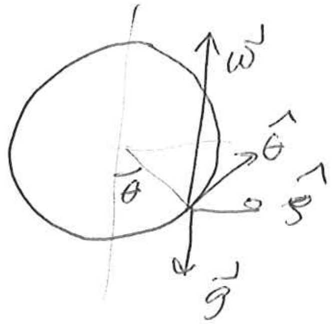
We want eqn of motion (7.69), in terms of θ. Use $\hat r, \hat\theta$ coords (polar) in the Non-Inertial frame.

$$\vec r = R\hat r, \quad \dot{\vec r} = R\dot{\hat r} = R\dot\theta\hat\theta = \vec v$$

$$\ddot{\vec r} = R\ddot\theta\hat\theta + R\dot\theta^2(-\hat r)$$

Take $\hat\theta$ component of ①

$$\Rightarrow \quad R\ddot\theta = \vec g\cdot\hat\theta + 2\vec v\cdot\vec\omega\times\hat\theta + \omega^2 R\sin\theta\,\hat\rho\cdot\hat\theta$$



$\vec g\cdot\hat\theta = -\sin\theta\, g$

$\vec\omega\times\hat\theta$ = into page

and $\vec v$ in page

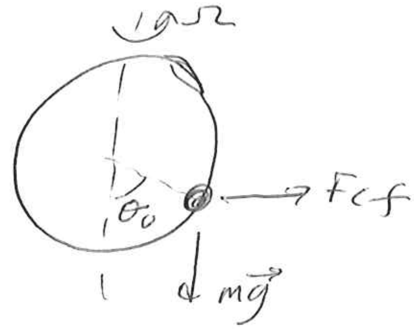
$\Rightarrow$ No coriolis

$\hat{\rho}\cdot\hat{\theta} = \cos\theta$

$\Rightarrow R\ddot{\theta} = -g\sin\theta + \omega^2 R\sin\theta\cos\theta$

$$\Rightarrow \boxed{\ddot{\theta} = -\frac{g}{R}\sin\theta + \omega^2\sin\theta\cos\theta}$$

agrees w (7.69)



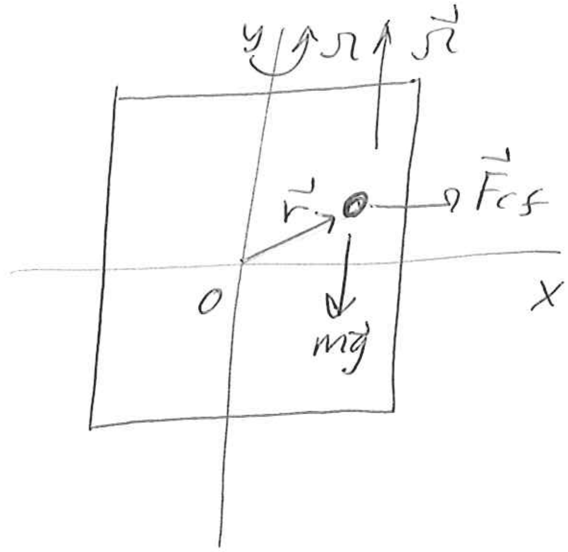
7.71 $\cos\theta_0 = (g/\omega^2 R)$

- explain in Frame of coil
- $\vec{V}=0$, so no $\vec{F}_{cor}$
- $\vec{F}_{cf}$ is outward, as shown
- $m\vec{g}$ is down
- Along $\hat{\theta}$, $mg\sin\theta_0 = m(\text{lever})\Omega^2\cos\theta_0$
  - lever $= R\sin\theta_0$

$\Rightarrow g\cancel{\sin\theta_0} = R\cancel{\sin\theta_0}\Omega^2\cos\theta_0$

$\Rightarrow \cos\theta_0 = \frac{g}{\Omega^2 R}$ ✓

9.18



$z = 0$
(constraint)

$$\ddot{\vec{r}} = \frac{\vec{F}}{m} + 2\vec{v}\times\vec{\Omega} + (\vec{\Omega}\times\vec{r})\times\vec{\Omega}$$

- $\vec{\Omega} = \hat{y}\,\Omega$
- $\vec{v}$ is in x-y plane $\Rightarrow$ $\vec{v}\times\vec{\Omega}$ is in or out of the plane $\Rightarrow$ does not act $\parallel$ to plane on mass. $\therefore$ irrelevant ($\vec{N}$ takes this up)
- take $\hat{x}$ & $\hat{y}$ components. Note that $(\vec{\Omega}\times\vec{r})$ points in, & $\vec{F}_{cf}$ points along x, with size $x\Omega^2$

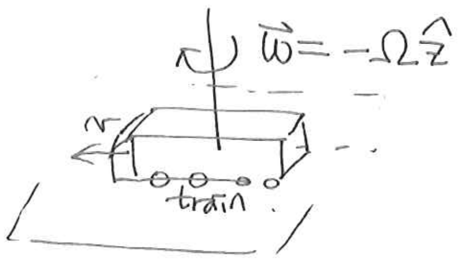
$$\therefore \boxed{\begin{aligned}\ddot{x} &= x\Omega^2 \\ \ddot{y} &= -g\end{aligned}} \qquad \begin{aligned} x(0) &= x_0 \\ y(0) &= 0 \end{aligned} \qquad \begin{aligned} \dot{x}(0) &= 0 \\ \dot{y}(0) &= 0 \end{aligned}$$

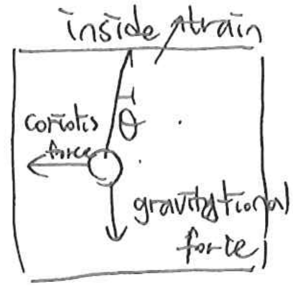
$$\Rightarrow \boxed{x = x_0\cosh(\Omega t),\quad y = -\tfrac{1}{2}gt^2}$$

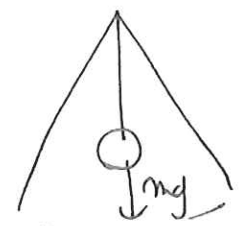
<u>Note</u> $\dot{y}(0) = 0$, $\dot{x}(0) = \Omega x_0 \sinh(\Omega t)\big|_0 = 0$

9.25

at the South pole







$v = 150\,m/s$

$$\tan\theta = \left(\frac{F_{gravitation}}{F_{Coriolis}}\right)^{-1} = \left(\frac{\cancel{m}g}{2\cancel{m}\omega v}\right)^{-1} = \left(\frac{9.8}{2\times\frac{2\pi}{24\times3600}\times150}\right)^{-1}$$

$$= \frac{2\times2\pi\times150}{24\times3600\times9.8} = 0.002225$$

$$\theta \simeq 0.002225 \text{ rad}$$

$$= \underline{0.127^\circ}.$$

the direction of deflection