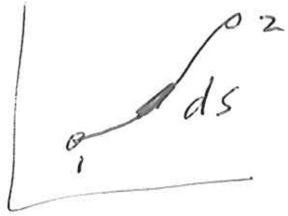
Phys 410 / F16

Solns PS 5

6.18



$$ds^2 = dr^2 + r^2 d\phi^2$$

$$L = \int_1^2 dr \sqrt{1 + r^2 \phi'^2} \qquad \phi' = \frac{d\phi}{dr}$$

$$L = L(\phi'; r)$$

$$\frac{\partial L}{\partial \phi} = 0 \qquad \frac{\partial L}{\partial \phi'} = \frac{r^2 \phi'}{\sqrt{}}$$

$$\frac{\partial L}{\partial \phi'} = \text{const}$$

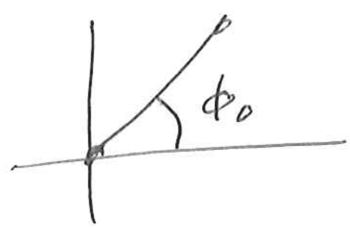
suppose point 1 is at (0,0)

$\Rightarrow$ const $= 0$ since $r = 0$

$$\Rightarrow \frac{r^2 \phi'}{\sqrt{}} = 0 \Rightarrow \phi' = 0$$

$$\Rightarrow \boxed{\phi = \phi_0}$$

$\Rightarrow$ straight line



7.1

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$$U = mgy$$

$$\mathcal{L} = T - U$$

$$\frac{\partial \mathcal{L}}{\partial x} = 0, \frac{\partial \mathcal{L}}{\partial z} = 0, \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}, \frac{\partial \mathcal{L}}{\partial \dot{z}} = m\dot{z}$$

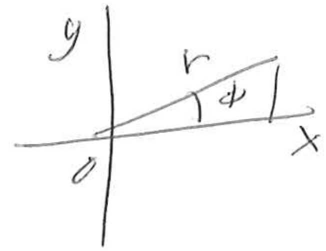
$$\Rightarrow \boxed{\begin{array}{l} m\ddot{x} = 0 \\ m\ddot{z} = 0 \end{array}}$$

$$\frac{\partial \mathcal{L}}{\partial y} = -mg \qquad \frac{\partial \mathcal{L}}{\partial \dot{y}} = m\dot{y}$$

$$\Rightarrow \boxed{m\ddot{y} = -mg}$$

7.3 2-D : $U(x,y) = \frac{1}{2} k r^2$

$r^2 = x^2 + y^2$



$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$

$\mathcal{L} = T - U$

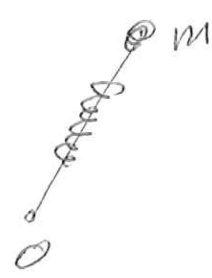
$\partial\mathcal{L}/\partial x = -kx, \quad \partial\mathcal{L}/\partial y = -ky$

$\partial\mathcal{L}/\partial \dot{x} = m\dot{x}, \quad \partial\mathcal{L}/\partial \dot{y} = m\dot{y}$

$\Rightarrow \quad m\ddot{x} = -kx$

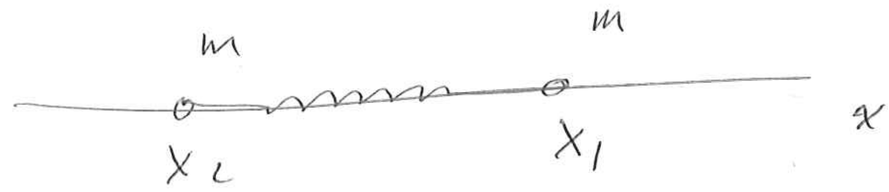
$\quad\quad m\ddot{y} = -ky$

$\Rightarrow \quad \boxed{m\ddot{\vec{r}} = -k\vec{r}}$



mass with
"spring force"
$\propto \vec{r}$, about origin, in 2-D

7-8



$$T = \frac{1}{2} m \left(\dot{x}_1^2 + \dot{x}_2^2\right)$$

$$U = \frac{1}{2} k x^2$$

$$x = x_1 - x_2 - l$$

(a) $\mathcal{L} = T - U = \mathcal{L}(\dot{x}_1, \dot{x}_2, x_1, x_2)$

(b) let $X \equiv \frac{1}{2}(x_1 + x_2)$

Now: $x_1 + x_2 = 2X$

$x_1 - x_2 = x + l$

$$\Rightarrow \quad 2x_1 = 2X + (x+l)$$

$$2x_2 = 2X - (x+l)$$

$$4\dot{x}_1^2 = 4\dot{X}^2 + \dot{x}^2 + 4\dot{X}\dot{x}$$

$$4\dot{x}_2^2 = 4\dot{X}^2 + \dot{x}^2 - 4\dot{X}\dot{x}$$

$$\Rightarrow \frac{\cancel{4}\left(\dot{x}_1^2 + \dot{x}_2^2\right)}{2} = \cancel{8}\dot{X}^2 + \frac{\cancel{2}\dot{x}^2}{4}$$

$$\Rightarrow \boxed{T = 2m\frac{\dot{X}^2}{2} + m\frac{\dot{x}^2}{4} \qquad U = \frac{1}{2} k x^2}$$

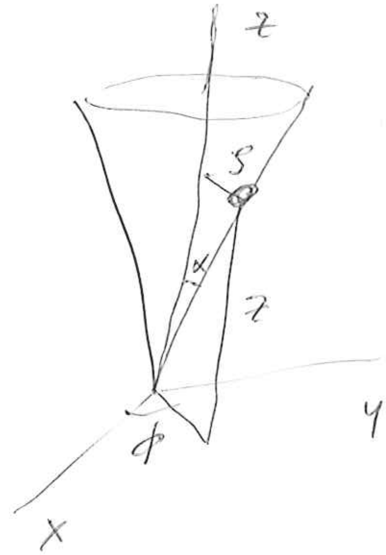
(c) $\partial\mathcal{L}/\partial X = 0 \Rightarrow \boxed{\dot{X} = \text{const}}$, vel of CM

$\partial\mathcal{L}/\partial x = -kx$, $\partial\mathcal{L}/\partial\dot{x} = \frac{1}{2}\dot{x} m$

$\Rightarrow \frac{m}{2}\ddot{x} = -kx$, $\boxed{m\ddot{x} = -2kx}$

$\Rightarrow$ HO

7.10 Cyl $\to \{\rho, \phi, z\}$



$x = \rho \cos\phi$

$y = \rho \sin\phi$

$z = \rho / \tan\alpha$

$\{\rho, \phi\}$ are generalized coords

$\{x, y, z\}$ Cartesian

one constraint

Also, $\rho^2 = x^2 + y^2$ $\{x, y\} \Rightarrow \{\rho, \phi\}$

$\tan\phi = y/x$