Solns PS4

Phys 410/F16

5.2H

See notes posted under "Misc."

5.42

$Q \equiv \omega_0 / 2\beta$

$e^{-\beta t}$ is damping

ω_0 is angular freq of osc

- Swings for many hours. Let decay time = 8 hr

$\Rightarrow \frac{1}{\beta} \sim 8$ hrs $= 8 \times 60 \times 60$ secs

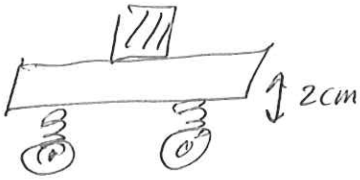
$l = 30\,m$, $\omega_0 = \sqrt{\frac{g}{l}} \sim \sqrt{\frac{10}{30}} = \sqrt{\frac{1}{3}}$

$Q = \sqrt{\frac{1}{3}} \, \frac{1}{2} \, 8 \times 60 \times 60$

$\boxed{Q \simeq 8300}$

$\underline{\underline{5.43}}$ (2π's may not be in right places)

(a)



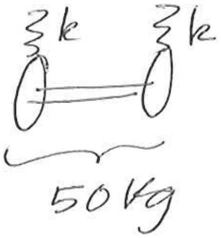
$$4Mg = 4k\Delta X$$

$$Mg = k\Delta X \Rightarrow k$$

$$M = 80$$

$$\boxed{k = 40{,}000}$$

(b)



$$M\omega^2 \Delta X \simeq 2k\Delta X$$

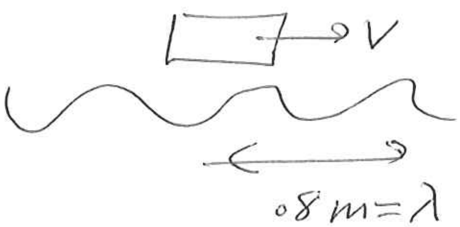
$$\Rightarrow \omega^2 \simeq \frac{2k}{M}$$

$$\omega = 2\pi f \Rightarrow f^2 = \frac{2k}{M_{50}} \frac{1}{(2\pi)^2}$$

$$M = 50$$

$$\Rightarrow f \quad \boxed{f \simeq 6.4/s}$$

(c)



$$\frac{V 2\pi}{\lambda} = 2\pi f$$

$$\Rightarrow V = \lambda f \Rightarrow V$$

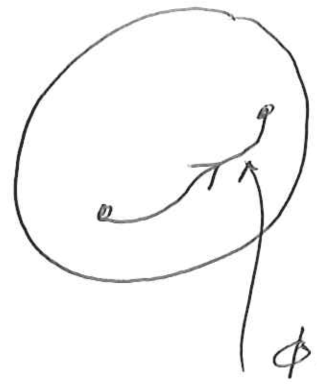
$$\text{so } V = \lambda f = \frac{\lambda}{2\pi}\left(\frac{2k}{M_{50}}\right)^{1/2} = \frac{\lambda}{2\pi}\left(\frac{2}{M_{50}}\right)^{1/2}\left(\frac{M_{80}\,g}{.02}\right)^{1/2}$$

$$\Rightarrow V = (.8)(6.4) \text{ m/s}$$

$$\boxed{V \simeq \frac{5\text{m}}{\text{s}} \sim 10 \text{ mp/h}}$$

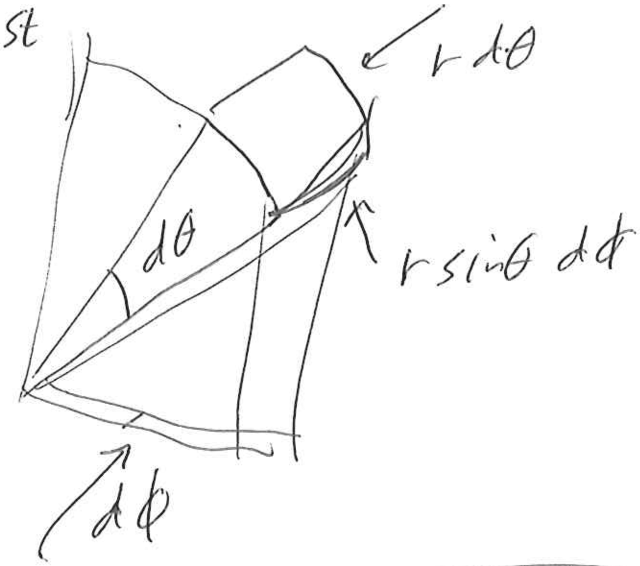
6.1 Shortest path between 2 points on a sphere

Use spherical coordinates



$\phi(\theta)$
$r = \text{const}$

$$ds^2\Big|_{\substack{\text{on surface}\\ r=\text{const}}} = (r\sin\theta\, d\phi)^2 + (r\, d\theta)^2$$



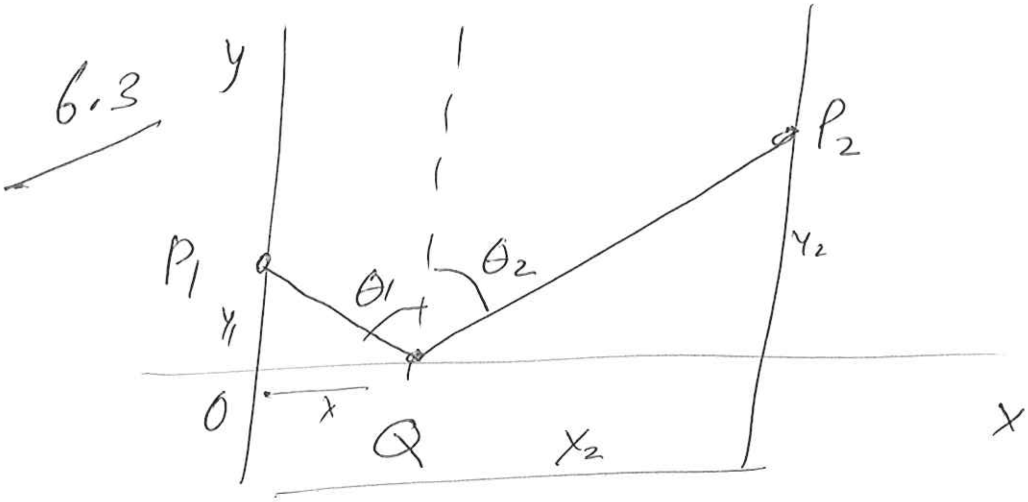
$$ds^2 = d\theta^2 r^2 \sqrt{1+\sin^2\theta\phi'^2}$$

$r = R$

$$\Rightarrow \boxed{\frac{L}{R} = \int_{\theta_1,\phi_1}^{\theta_2,\phi_2} d\theta \sqrt{1+\sin^2\theta\, \phi'(\theta)^2}}$$

Note $L = L(\phi';\theta)$ $\quad L \neq L(\phi)$

6.3



$P_1 = (0, y_1, 0)$ $\quad$ y_1 given

$P_2 = (x_2, y_2, 0)$ $\quad$ x_2, y_2 given

$Q = (x, 0, z)$ $\quad$ x, z unknown

- For both rays, speed = const

$\Rightarrow$ straight paths,

$\Rightarrow$ travel time $= \dfrac{\text{Length of path}}{c}$

- $L_1^2 = x^2 + y_1^2 + z^2$

$L_2^2 = (x_2 - x)^2 + y_2^2 + z^2$

$T = \dfrac{L_1}{c} + \dfrac{L_2}{c}$

Note $T = T(x, z)$

Need $\dfrac{\partial T}{\partial x} = 0, \dfrac{\partial T}{\partial z} = 0$

$$\frac{\partial T}{\partial z} = \frac{z}{L_1 c} + \frac{z}{L_2 c} = 0$$

$$\Rightarrow \boxed{z = 0}$$

$$\frac{\partial T}{\partial x} = \frac{x}{L_1 c} - \frac{(x_2 - x)}{L_2 c} = 0$$

For $z = 0$

$$\frac{x}{L_1} = \sin\theta_1, \quad \frac{x_2 - x}{L_2} = \sin\theta_2$$

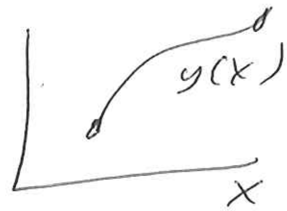
$$\Rightarrow \boxed{\begin{gathered} \sin\theta_1 = \sin\theta_2 \\ \theta_1 = \theta_2 \end{gathered}} \qquad \boxed{\text{also } z = 0}$$

Snell's law

6.9

$$S = \int_{0,0}^{1,1} (y'^2 + yy' + y^2)\,dx$$

Make S stationary for some $y(x)$.



$$\frac{\delta S}{\delta Y} = 0 \Rightarrow \frac{\partial f}{\partial Y} = \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)$$

$$\frac{\partial f}{\partial y} = y' + 2y \quad , \quad \frac{\partial f}{\partial y'} = 2y' + y$$

$$\Rightarrow \quad 2y'' + y' = y' + 2y$$

$$\Rightarrow \quad y'' = y \quad \Rightarrow y \sim \left\{ \begin{matrix} \sinh x \\ \cosh x \end{matrix} \right\}$$

$$\Rightarrow \quad \boxed{y = \frac{\sinh(x)}{\sinh(1)}} \qquad \begin{matrix} y(0) = 0 \\ y(1) = 1 \end{matrix}$$

6.10

$$S = \int_1^2 dx\, f(y', y, x)$$

Suppose $f = f(y', x)$,

ie indep of y

Then $\frac{\partial f}{\partial y} = \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)$

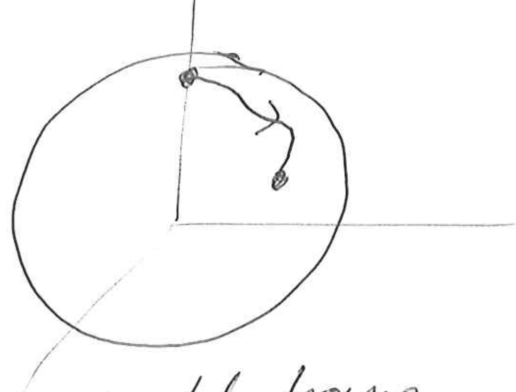
$$\Rightarrow \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

$$\Rightarrow \boxed{\frac{\partial f}{\partial y'} = \text{const}}$$ + apply B.C.'s

$\Rightarrow$ one integral done easily

6.16 The sphere is completely symmetric, ie, we can pick $\theta_1 = 0$ and (θ_2, ϕ_2) as then end pt.

ie we let the z-axis cutting sphere, the North Pole, as starting point. No generality is lost.



Now we go from North pole to θ_2, ϕ_2.

Might as well pick $\phi_2 = 0$.

Also, $\theta = 0$ is a point which could have any ϕ_1. So pick $\phi_1 = 0$.

$\therefore$ Minimize $\frac{L}{R} = \int_0^{\theta_2} d\theta \sqrt{1+\sin^2\theta\, \phi'(\theta)^2}$

$$\frac{\partial L}{\partial \phi} = 0 \quad \frac{\partial L}{\partial \phi'} = \frac{\phi' \sin^2\theta}{\sqrt{1+\sin^2\theta\, \phi'^2}}$$

so $\frac{\partial L}{\partial \phi} = \frac{d}{d\theta}\frac{\partial L}{\partial \phi'}$ is E-L $\Rightarrow \frac{\partial L}{\partial \phi'} = $ const

$$\Rightarrow \quad \frac{\phi' \sin^2\theta}{\sqrt{1+\sin^2\theta\,\phi'^2}} = \text{const}$$

This must be true for all $\phi(\theta)$

But also true at $\theta = 0 \Rightarrow \left.\dfrac{\phi' \sin^2\theta}{\sqrt{1+\sin^2\theta\,\phi'^2}}\right|_{\theta=0} = \text{const}$

$$\Rightarrow \quad \text{const} = 0$$

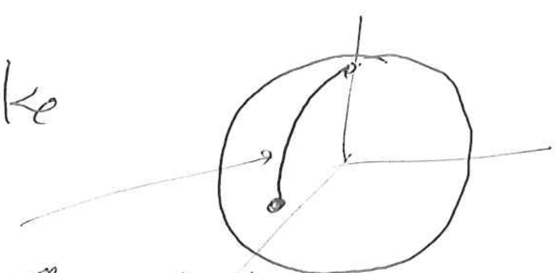
$$\Rightarrow \quad \frac{\phi' \sin^2\theta}{\sqrt{1+\sin^2\theta\,\phi'^2}} = 0 \quad \forall\, \theta$$

$$\Rightarrow \phi' = 0 \quad \forall\, \theta$$

$$\Rightarrow \phi = \text{const}$$

$\phi(0) = 0 \quad \Rightarrow \quad \boxed{\phi = 0}$ from $\theta = 0$ to θ_2

This looks like



$\phi = 0$ coordinate

$\theta = [0, \theta_2]$

Its along a longitude, ie, a great circle. $\leftarrow$ Path of shortest distance