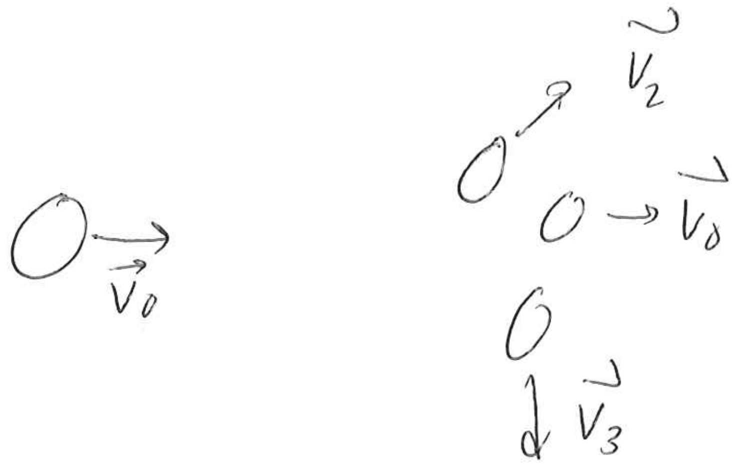
Phy 410 / F16

PS2 Solns

3.3



$$\vec{V_2} \cdot \vec{V_3} = 0 \quad , V_2 = V_3$$

$$3\vec{V_0} = \vec{V_0} + \vec{V_2} + \vec{V_3}$$

$$\Rightarrow 2\vec{V_0} = \vec{V_2} + \vec{V_3} \qquad (1)$$

$$4V_0^2 = V_2^2 + V_3^2 + 2V_2 \cdot V_3$$

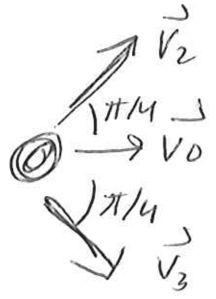
$$= 2V_2^2 \Rightarrow \boxed{\sqrt{2}V_0 = V_2 = V_3}$$

From (1), $2\vec{V_0} \cdot \vec{V_2} = V_2^2 = 2V_0^2$ $\qquad 2\vec{V_0} =$

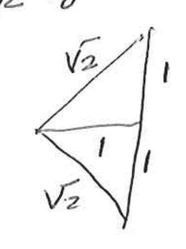
$$\vec{V_0} \cdot \vec{V_2} = V_0^2$$

$$V_2 V_0 \cos\theta = V_0^2$$

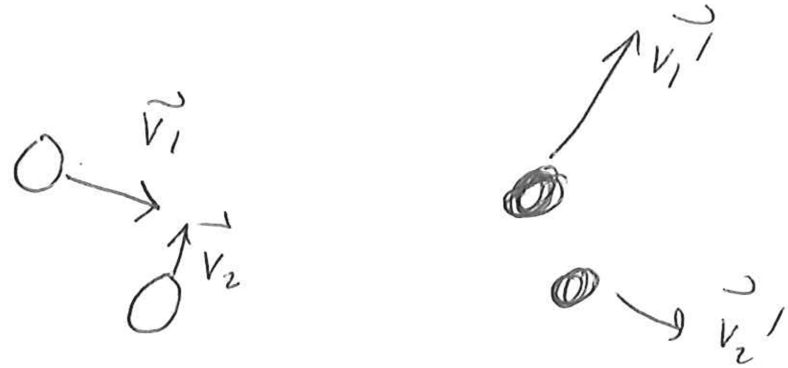
$$\cos\theta = \frac{V_0}{V_2} = \frac{V_0}{\sqrt{2}V_0} = \frac{1}{\sqrt{2}} = \pi/4$$



$V_2 = V_3 = \sqrt{2}V_0$



3.5

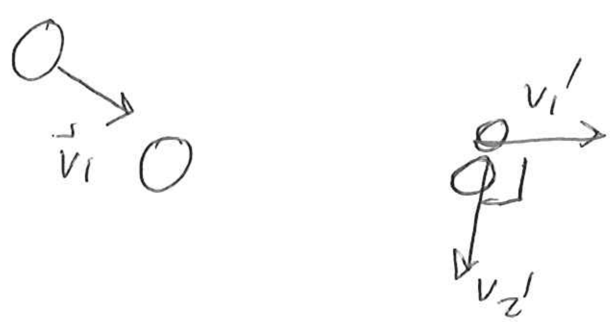


① $\vec{v}_1+\vec{v}_2=\vec{v}_1'+\vec{v}_2'$ Momentum

② $v_1^2+v_2^2=v_1'^2+v_2'^2$ KE

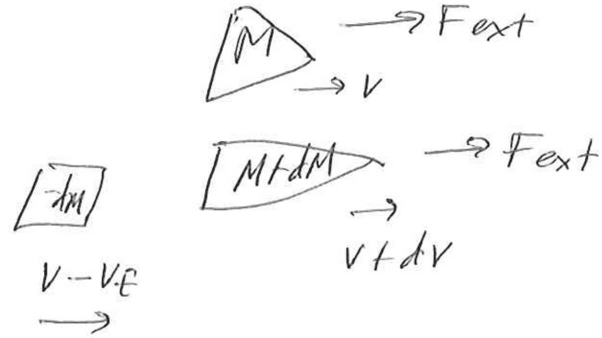
① $\Rightarrow$ $\cancel{v_1^2}+\cancel{v_2^2}+2v_1\cdot v_2=\cancel{v_1'^2}+\cancel{v_2'^2}+2v_1'\cdot v_2'$

$\vec{v}_2=0 \Rightarrow \boxed{0=\vec{v}_1'\cdot\vec{v}_2'}$



3.11 (a), (b), (d) only

(a)



$$\frac{d}{dt}(MV) = F_{ext}$$

$$\Delta(MV) = F_{ext}\,\Delta t$$

Before $\quad MV = Mv$

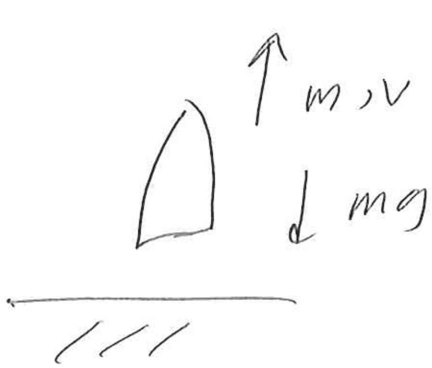
After $\quad MV = (M+dM)(v+dv) + (-dM)(v-v_E)$

$$\therefore \Delta(MV) \rightarrow M dv + \cancel{dMv} - \cancel{dMv} + dMv_E$$

$$\Rightarrow M dv + dM v_E = F_{ext}\, dt$$

$$\Rightarrow \boxed{M\dot{v} + \dot{M} v_E = F_{ext}}$$

(b)



$$\boxed{m\dot{v} = -\dot{m} v_E - mg}$$

* assumes rocket off ground ie, $N=0$

$$\dot{m} = -k \Rightarrow m = m_0 - kt > 0$$

$$\Rightarrow \dot{v} = \frac{-\cancel{k} v_E}{m_0 - kt} - g = fn(t)$$

$$\Rightarrow v(t) = +\cancel{k} v_E \ln(m_0 - kt) - gt + c$$

$$v(0) = 0 \Rightarrow c = -v_E \ln m_0$$

$$\Rightarrow \boxed{v(t) = v_E \ln\left(1 - \frac{kt}{m_0}\right) - gt}$$

for $m_0 > kt$

Check $\dot{v} = \frac{-v_E(k/m_0)}{1 - kt/m_0} - g$ ✓

(d) ~~ck~~ $m\dot{v} = -\dot{m} v_E - mg$

$m\dot{v} = k v_E - mg$, if $N = 0$

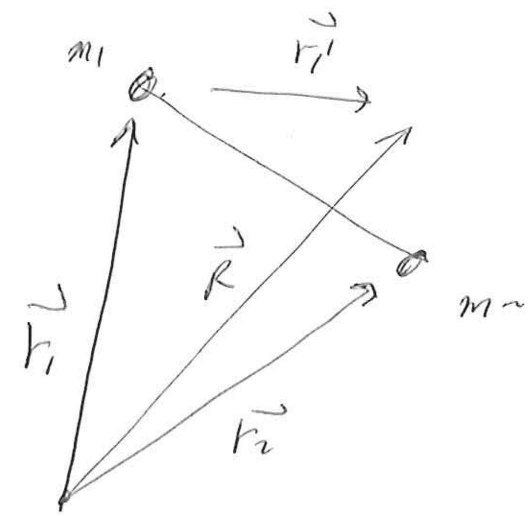
if $k v_E < g m_0 \Rightarrow \dot{v} < 0$ at $t = 0$

Not possible since $N = 0$

$\therefore$ stays on ground until $m(t) \downarrow$

$\Rightarrow k v_E \geq g m(t)$.

3.18



$$\left(\sum_k m_k\right) \vec{R} \equiv \sum_k (m_k \vec{r}_k)$$

Let $\vec{r_1}' = \vec{R} - \vec{r_1}$. Let $\vec{d} = \vec{r_2} - \vec{r_1}$

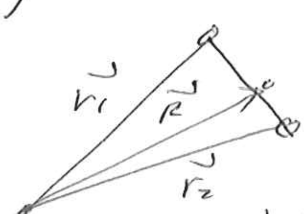
(a) Consider $\vec{r_1}' \times \vec{d}$.

$$\vec{r_1}' \times \vec{d} = (\vec{R} - \vec{r_1}) \times (\vec{r_2} - \vec{r_1})$$

$$= (\vec{R} \times \vec{r_2}) - (\vec{R} \times \vec{r_1}) - (\vec{r_1} \times \vec{r_2})$$

$$= \frac{m_1 \vec{r_1} \times \vec{r_2}}{\sum m} - \frac{m_2 \vec{r_2} \times \vec{r_1}}{\sum m} - (\vec{r_1} \times \vec{r_2})$$

$$= \frac{\sum m}{\sum m} (\vec{r_1} \times \vec{r_2}) - (\vec{r_1} \times \vec{r_2})$$



$$= 0$$

(b) Consider $m_1 \vec{r_1}' + m_2 \vec{r_2}' = m_1 (\vec{R} - \vec{r_1}) + m_2 (\vec{R} - \vec{r_2})$

$= (\sum m) \vec{R} - \sum (m \vec{r}) = 0 \Rightarrow m_1 \vec{r_1}' = -m_2 \vec{r_2}'$.

But $\vec{r_1}' // \vec{r_2}'$, and opposed $\Rightarrow$ $m_1 r_1' = m_2 r_2'$

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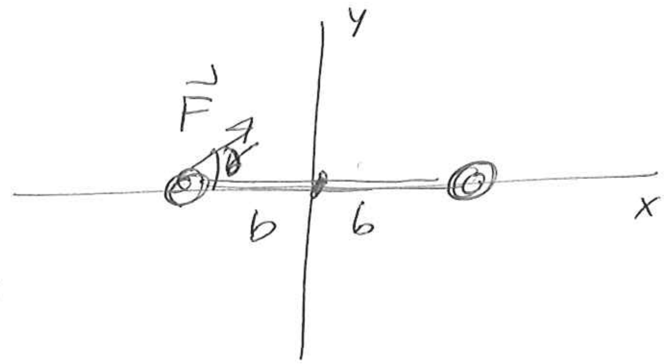


Impulse $\vec{F}\Delta t$

$\Rightarrow \Delta \vec{V}_{CM}$

and $\Delta \vec{L}$

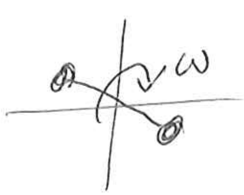
Center of Mass ~~MCP~~ $(\text{Mass})\, \dot{\vec{V}}_{CM} = \vec{F}$

$$\Rightarrow \boxed{\Delta \vec{V}_{CM} = \vec{F}\Delta t / 2m}$$

Angular mom about CM

$$\dot{\vec{L}}\ (\text{about CM}) = (\vec{r} \times \vec{F})(\text{about CM})$$

all vectors along z



$$\Delta L = m b^2 \Delta\omega + m b^2 \Delta\omega$$

$$\Delta L = F \sin\gamma\, b\, \Delta t$$

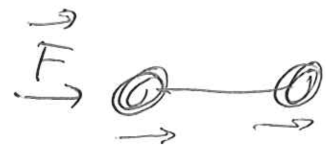
$$\Rightarrow \boxed{2mb\Delta\omega = F\Delta t \sin\gamma}$$

Find $\vec{V}_{left}$ and $\vec{V}_{right}$

$$\vec{v}_{left} = \frac{\vec{F}\Delta t}{2m} + \hat{y}\,\frac{F\Delta t \sin\gamma}{2m}$$

$$\vec{v}_{right} = \frac{\vec{F}\Delta t}{m} - \hat{y}\,\frac{F\Delta t \sin\gamma}{2m}$$

Check $\gamma = 0$



$$\Rightarrow \quad \vec{v}_{left} = \vec{F}\Delta t / 2m = \vec{v}_{right} \ \#$$

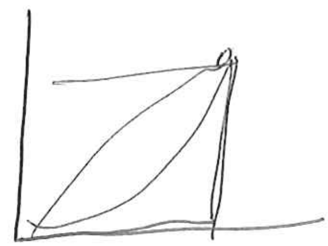
Check $\gamma = \pi/2$

$\vec{v}_{left}$ = up , $\vec{v}_{right}$ = up or down

$$v_{left,up} = \frac{F\Delta t}{2m} + \frac{F\Delta t}{2m} = \frac{F\Delta t}{m}$$

$$v_{right,up/down} = \frac{F\Delta t}{2m} - \frac{F\Delta t}{2m} = 0$$

4.2 $\vec{F} = (x^2, 2xy)$



(a) $\int_{\rightarrow} F\cdot dr = \int_0^1 x^2\,dx = \frac{1}{3}$

$\int_{\uparrow} \vec{F}\cdot d\vec{r} = \int_0^1 dy\, 2yx\big|_1 = 1$

$\Rightarrow \boxed{\int = 4/3}$

(b) $y = x^2$ $\quad \int F\cdot dr = \int_0^1 dx\, x^2 + \int dy\, 2xy$

$dy = 2x\,dx$ $\quad = \frac{1}{3} + \int_0^1 \cancel{dy\,2y}\, 2x\,dx\; 2x^2 x$

$= \frac{1}{3} + 4\int_0^1 dx\, x^4 = \frac{1}{3} + \frac{4}{5} = \boxed{\frac{17}{15}}$

(c) $x = t^3,\ y = t^2,\ dx = 3t^2\,dt,\ dy = 2t\,dt$

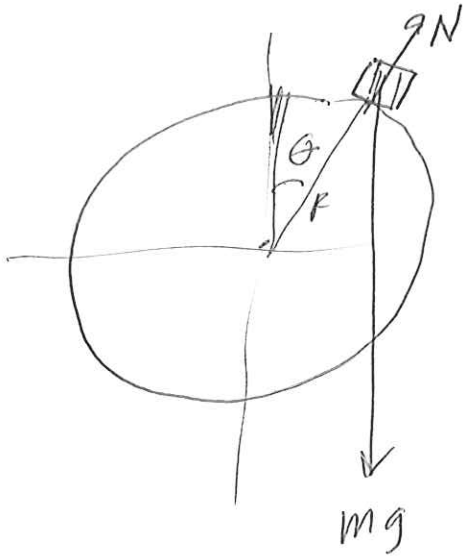
$\int F\cdot dr = \int_0^1 dx\, x^2 + \int dy\, 2xy$

$= \frac{1}{3} + \int_0^1 2t\,dt\; 2\,t^3 t^2$

$= \frac{1}{3} + 4\int_0^1 dt\, t^6 = \frac{1}{3} + \frac{4}{7} = \boxed{\frac{19}{21}}$

Note $\vec{\nabla}\times\vec{F} = 2y - 0 = 2y \neq 0$

4.8



- Assume $N \neq 0$
- Puck does not accel along $N \Rightarrow N = mg\cos\theta$

$\Rightarrow mg\cos\theta - N = mv^2/R$

- N does no work

$\Rightarrow$ all potential $\mathcal{E}$ from mg

- let $U = -mg(R - R\cos\theta)$. $\quad U(\theta=0)=0$

$U(\theta=\pi/2) = -mgR$

$(-)$ sign needed since PE gets lower.

use $K + U = (K+U)_0 = U(0) = 0$

$$K = \frac{1}{2}mR^2\dot{\theta}^2$$

$$\Rightarrow \frac{1}{2}mR^2\dot{\theta}^2 = mgR(1-\cos\theta)$$

- For N, $\quad mg\cos\theta - N = mR\dot{\theta}^2 \quad — ①$
- Puck leaves sphere $\Rightarrow N = 0$

$$\Rightarrow g\cos\theta = R\dot{\theta}^2\Big|_{leave}$$

Plug $\dot{\theta}^2\Big|_{leave}$ into ① $\Rightarrow$

$$\frac{1}{2}R\dot{\theta}^2\Big|_{leave} = g(1-\cos\theta)$$

$$\frac{1}{2}\cos\theta = 1 - \cos\theta \Rightarrow \frac{3}{2}\cos\theta = 1$$

$$\boxed{\cos\theta = 2/3}$$

4.12 (a) $f = x^2 + z^3$

$$\vec{\nabla} f = 2x\hat{x} + 3z^2\hat{z}$$

(b) $f = ky$, $\vec{\nabla} f = k\hat{y}$

(c) $f = r = \sqrt{x^2+y^2+z^2}$

$$\frac{\partial f}{\partial x} = \left.\frac{\partial r}{\partial x}\right)_{y,z} = \frac{\frac{1}{2}\,\cancel{2}x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$$

$$\Rightarrow \vec{\nabla} f = \frac{1}{r}(x\hat{x} + y\hat{y} + z\hat{z})$$

$$\Rightarrow \vec{\nabla} f = \frac{\vec{r}}{r} = \hat{r}$$

(d) $f = 1/r$

$$\vec{\nabla} f = -\frac{1}{r^2}\vec{\nabla} r = -\frac{\hat{r}}{r^2}$$, using (c)

4.13 (a) $f = \ln r$

$$\vec{\nabla} f = \frac{1}{r} \vec{\nabla} r = \frac{\hat{r}}{r}$$

(c) $f = g(r)$

$$\vec{\nabla} f = g'(r) \vec{\nabla} r = g'(r) \hat{r}$$

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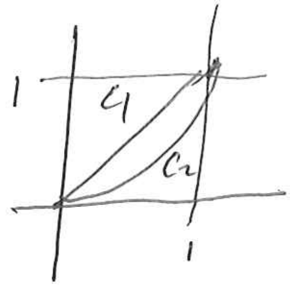

$C_1 \Rightarrow y = x$

$C_2 \Rightarrow y = x^2$

(a) $\vec{F} = (y, x)$ $\qquad \vec{\nabla} \times \vec{F} = 1 - 1 = 0$

$$\int_{C_1} \vec{F} \cdot d\vec{r} = \int dx\, y + \int dy\, x = 2\int dx\, x = \boxed{1}$$

$$\int_{C_2} \vec{F} \cdot d\vec{r} = \int dx\, y + \int dy\, x = \int_0^1 dx\, x^2 + \int_0^1 2x\, dx\, x$$

$$= \frac{1}{3} + 2\frac{1}{3} = \boxed{1}$$

(b) $\vec{F} = (-y, x)$ $\qquad \vec{\nabla} \times \vec{F} = \partial_x x - \partial_y (-y) = 2 \neq 0$

$$\int_{C_1} \vec{F} \cdot d\vec{r} = -\int dx\, y + \int dy\, x$$

$$= -\int_0^1 dx\, x + \int_0^1 dy\, y = \boxed{0}$$

$$\int_{C_2} \vec{F} \cdot d\vec{r} = -\int dx\, y + \int dy\, x$$

$$= -\int_0^1 dx\, x^2 + \int_0^1 2x\, dx\, x$$

$$= -\frac{1}{3} + 2\frac{1}{3} = \boxed{1/3}$$