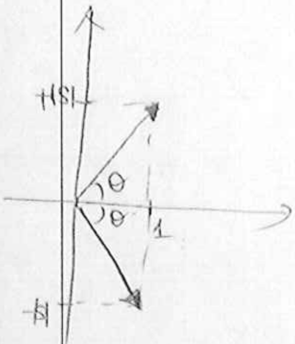
# Phys 410, F16

# Solns PS1

1.6.

$\vec{b} = \hat{x} + s\hat{y}$ , $\vec{c} = \hat{x} - s\hat{y}$

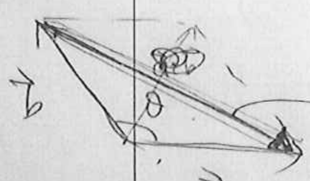
$\vec{b}\cdot\vec{c} = 1 - s^2 = 0$ where $s = \pm 1$.




$$\boxed{2\theta = \frac{\pi}{2} \Leftarrow \vec{b}\cdot\vec{c} = 0}$$

$\theta = \frac{\pi}{4} \Rightarrow |s| = 1$ or $s = \pm 1$

1.9.




$\vec{c} = \vec{a} - \vec{b}$

$$|\vec{c}|^2 = |\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) = a^2 + b^2 \underbrace{-2\vec{a}\cdot\vec{b}}_{= -2ab\cos\theta}$$

$\therefore c^2 = a^2 + b^2 - 2ab\cos\theta$

1.17

let $\vec{r} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z}$

$\vec{u} = u_x\hat{x} + u_y\hat{y} + u_z\hat{z}$

$\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$

$$\begin{aligned}
\vec{r}\times(\vec{u}+\vec{v}) &= \{r_y(u_z+v_z) - r_z(u_y+v_y)\}\hat{x} \\
&\quad + \{r_z(u_x+v_x) - r_x(u_z+v_z)\}\hat{y} \\
&\quad + \{r_x(u_y+v_y) - r_y(u_x+v_x)\}\hat{z} \\
&= (r_y u_z - r_z u_y)\hat{x} + (r_z u_x - r_x u_z)\hat{y} + (r_x u_y - r_y u_x)\hat{z} \\
&\quad + (r_y v_z - r_z v_y)\hat{x} + (r_z v_x - r_x v_z)\hat{y} + (r_x v_y - r_y v_x)\hat{z} \\
&= (\vec{r}\times\vec{u}) + (\vec{r}\times\vec{v})
\end{aligned}$$

(b)

$$\frac{d}{dt}(\vec{r}\times\vec{s}) = \frac{d}{dt}\left\{(r_y s_z - r_z s_y)\hat{x} + (r_z s_x - r_x s_z)\hat{y} + (r_x s_y - r_y s_x)\hat{z}\right\}$$

$$= \left(\frac{dr_y}{dt}s_z + r_y\frac{ds_z}{dt} - \frac{dr_z}{dt}s_y - r_z\frac{ds_y}{dt}\right)\hat{x}$$

$$+ \left(\frac{dr_z}{dt}s_x + r_z\frac{ds_x}{dt} - \frac{dr_x}{dt}s_z - r_x\frac{ds_z}{dt}\right)\hat{y}$$

$$+ \left(\frac{dr_x}{dt}s_y + r_x\frac{ds_y}{dt} - \frac{dr_y}{dt}s_x - r_y\frac{ds_x}{dt}\right)\hat{z}$$

$$= \frac{d\vec{r}}{dt}\times\vec{s} + \vec{r}\times\frac{d\vec{s}}{dt}$$

1.23

$$\vec{b}\cdot\vec{v} = \lambda, \quad \vec{b}\times\vec{v} = \vec{c}$$

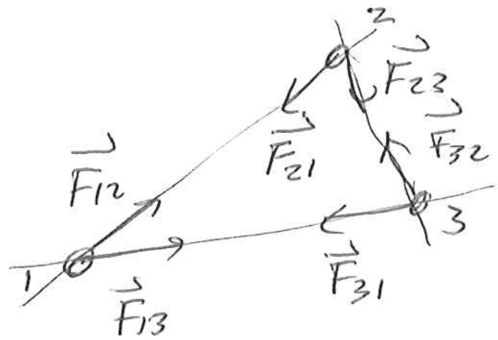
$$\vec{b}\times\underbrace{(\vec{b}\times\vec{v})}_{\vec{c}} = \vec{b}\underbrace{(\vec{b}\cdot\vec{v})}_{\lambda} - \vec{v}\underbrace{(\vec{b}\cdot\vec{b})}_{b^2}.$$

$$\therefore \quad \lambda\vec{b} - \vec{b}\times\vec{c} = b^2\vec{v}$$

$$\therefore \quad \vec{v} = \frac{\lambda\vec{b} - \vec{b}\times\vec{c}}{b^2}$$

## 1.28 3 particles

we have



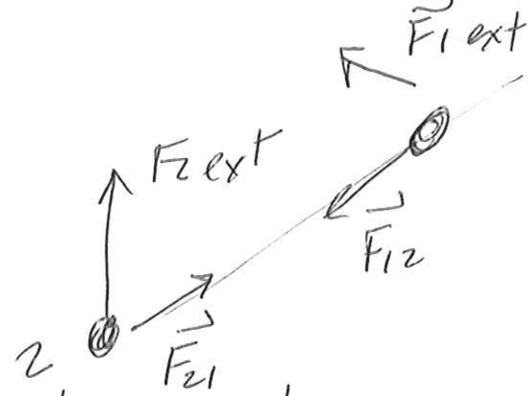

$$\dot{\vec p}_1 = \vec F_{12} + \vec F_{13}, \quad \dot{\vec p}_2 = \vec F_{21} + \vec F_{23}, \quad \dot{\vec p}_3 = \vec F_{31} + \vec F_{32}$$

$$(\vec p_1 + \vec p_2 + \vec p_3)^{\cdot} = \vec F_{12} + \vec F_{13} + \vec F_{21} + \vec F_{23} + \vec F_{31} + \vec F_{32}$$

equal

$$\Rightarrow \quad (\vec p_1 + \vec p_2 + \vec p_3)^{\cdot} = 0$$

1031




① 3d Law ⇒ momentum conserved

$$\dot{\vec{p}}_1 = \vec{F}_{12} + \vec{F}_1 ext$$

$$\dot{\vec{p}}_2 = \vec{F}_{21} + \vec{F}_2 ext$$

$$\dot{\vec{p}}_1 + \dot{\vec{p}}_2 = \vec{F}_1 ext + \vec{F}_2 ext$$

$$\Rightarrow \quad \dot{\vec{P}} = \vec{F}ext = (\vec{F}_1 + \vec{F}_2)ext$$

$$\vec{P} \equiv \vec{p}_1 + \vec{p}_2$$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$\therefore \vec{F}ext = 0 \text{ and }$$

$$\Rightarrow \boxed{\dot{\vec{P}} = 0}$$

② Momentum conserved ⇒ 3d Law

$$\dot{\vec{p}}_1 = \vec{F}_{12}, \quad \dot{\vec{p}}_2 = \vec{F}_{21}$$

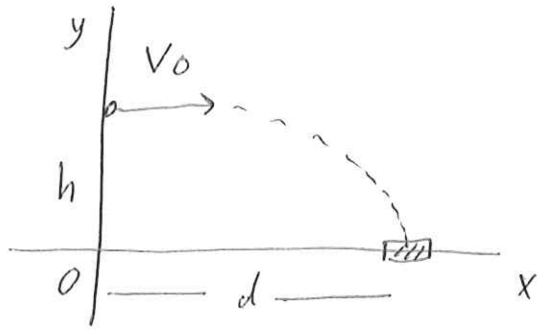
$$\dot{\vec{p}}_1 + \dot{\vec{p}}_2 = \vec{F}_{12} + \vec{F}_{21}$$

Momentum conserved for no $\vec{F}ext$ ⇒ $\vec{p}_1 + \vec{p}_2 = const$

$$\Rightarrow 0 = \vec{F}_{12} + \vec{F}_{21}$$

⇒ 3d Law

## 1.36




(a) $\vec{a} = -\hat{y} g$ $\qquad$ $\ddot{x} = 0$

$\ddot{y} = -g$

$\dot{x} = v_0, \quad \dot{y} = -gt$ $\qquad$ $\dot{x}(0) = v_0$

$\dot{y}(0) = 0$

$x(t) = v_0 t, \quad y(t) = h - \frac{gt^2}{2}$ $\qquad$ $x(0) = 0$

$y(0) = h$

(b) Let rabt be @ distance $d$ at $t = 0$

want $t \ni y = 0$ and $x = d$

$\Rightarrow d = v_0 t, \quad 0 = h - gt^2/2$

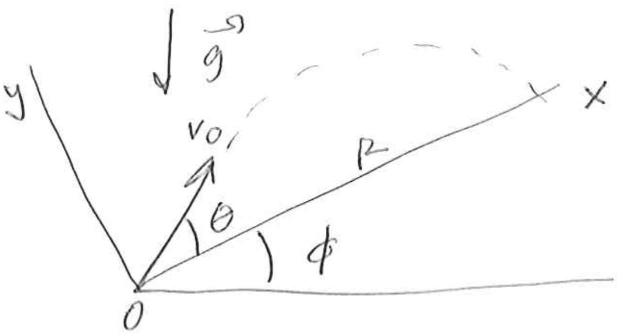
eliminate $t \Rightarrow h = \frac{g}{2} \frac{d^2}{v_0^2}$

$\therefore \boxed{d = \left(\frac{2 v_0^2 h}{g}\right)^{1/2}}$ $\qquad$ $\Rightarrow d = 224\,m$

$v_0 = 50, h = 100, g = 10$

(c) $\pm 10\,m$ is small compared to $224\,m$. $\therefore$ lets approximate. $\Delta d = v_0 \Delta t$; thus $\Delta t = \frac{\Delta d}{v_0}$, exactly dont need to approximate. $\Rightarrow \Delta t = \frac{10}{50} = \frac{1}{5}$ sec

## 1.39




$\ddot{\vec r} = +\vec g$

use ${}^{y}\llcorner_{x}$ as given

$\ddot x = +\vec g\cdot\hat x$ , $\ddot y = +\vec g\cdot\hat y$ $\quad \vec g = -g\downarrow$

$\vec g\cdot\hat x = -g\,\overset{\sin}{\cancel{\cos}}\,\phi$ , $\vec g\cdot\hat y = -g\,\overset{\cos}{\cancel{\sin}}\,\phi$

Check $\phi = 0 \Rightarrow {}^{y}\llcorner_{x}\ \downarrow\vec g \Rightarrow \vec g = -g\hat y$

$\Rightarrow \ddot x = -g\sin\phi$, $\ddot y = -g\cos\phi$

$\dot x = V_0\cos\theta - g\sin\phi\, t$ , $\dot y = V_0\sin\theta - g\cos\phi\, t$

$x = V_0\cos\theta\, t - g\sin\phi\,\frac{t^2}{2}$ , $y = V_0\sin\theta\, t - g\cos\phi\,\frac{t^2}{2}$

Checks for $\phi = 0$, $\theta = 0$, though $y \geq 0$ condition

We want R up the plane in x $\ni y = 0$

$\Rightarrow v_0\sin\theta = g\cos\phi\, t/2$

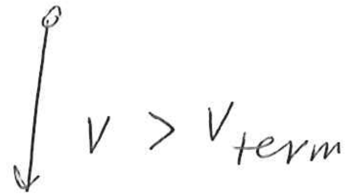
$$\Rightarrow R = \frac{v_0^2\cos\theta\, 2\sin\theta}{g\cos\phi} - \cancel{g}\frac{\sin\phi}{\cancel{2}}\,\frac{\overset{2}{\cancel{4}}\, v_0^2\sin^2\theta}{g^{\cancel{2}}\cos^2\phi}$$

$$R = \frac{2v_0^2\sin\theta}{g\cos^2\phi}\left[\cos\theta\cos\phi - \sin\theta\sin\phi\right]$$

$dR/d\theta = 0$
$\Rightarrow \cos\theta\cos(\;)$
$= \sin\theta\sin(\;)$

$$\boxed{R = \frac{2v_0^2\sin\theta}{g\cos^2\phi}\cos(\theta+\phi)}$$

## 2.5



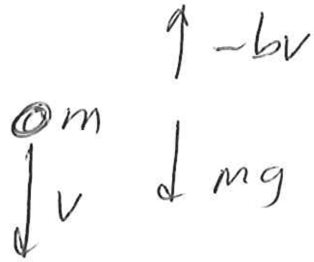

• Note that $v > v_{term}$

$\Rightarrow$ $f$ stronger than $mg$

$\Rightarrow$ $v(t)$ must slow down

• Let $y$ axis increase vertically down

$\Rightarrow$




$\Rightarrow$ $m\dot{v} = mg - bv$

$v(0) = v_0$

$$\boxed{\dot{v} = g - v/\tau}$$

• Let $\boxed{u = v - g\tau}$ $\Rightarrow \tau\dot{u} = -u$

$\Rightarrow$ $u = u(0)e^{-t/\tau}$

$u(0) = v(0) - g\tau = v_0 - g\tau$

$\Rightarrow$ $u = (v_0 - g\tau)e^{-t/\tau}$

$$\Rightarrow \boxed{v(t) = (v_0 - g\tau)e^{-t/\tau} + g\tau}$$

$t = 0$, $v = v_0$. $t \to \infty$, $v \to g(\tau)$

else $v > g\tau \ \forall t$

2.5 (repeat) assume $v_0 > g\tau$

ie $v > g\tau$

$$\dot{v} = g - v/\tau$$

$$\Rightarrow \frac{dv}{g - v/\tau} = dt$$

rewrite $\dfrac{dv}{v - g\tau} = -\dfrac{dt}{\tau}$ , $v > g\tau$

integrate $\Rightarrow \ln(v - g\tau) = -\dfrac{t}{\tau} + C$

where $v > g\tau$

$t = 0 \Rightarrow \ln(v_0 - g\tau) = C$

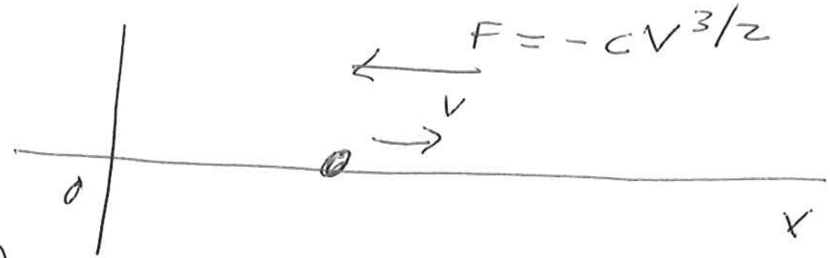
$$\Rightarrow \ln\left(\frac{v - g\tau}{v_0 - g\tau}\right) = -t/\tau$$

$$\Rightarrow \frac{v - g\tau}{v_0 - g\tau} = e^{-t/\tau}$$

$$\Rightarrow \boxed{v = g\tau + (v_0 - g\tau)e^{-t/\tau}}$$

agrees

2.8




~~Let~~ $v(0) = v_0$
use sep var on ODE

$$m\dot{v} = -cv^{3/2}$$

For convenience, let $c \to c'm$

$$\dot{v} = -c'v^{3/2}$$

separable first order ODE

$$\frac{dv}{v^{3/2}} = -c'dt$$

$$d\left(1/v^{1/2}\right) = -\frac{1}{v^{3/2}}\frac{1}{2}$$

$$\Rightarrow \int_{v_0}^{v} d\left(\frac{2}{v^{1/2}}\right) = c'\int_0^t dt'$$

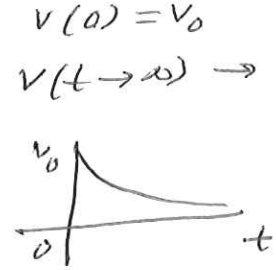
$$\Rightarrow \left(\frac{1}{v^{1/2}} - \frac{1}{v_0^{1/2}}\right) = \frac{c't}{2}$$

$$\frac{1}{v^{1/2}} = \frac{1}{v_0^{1/2}} + \frac{c't}{2} \qquad \frac{v^{1/2}}{v_0^{1/2}} = \frac{1}{1 + \frac{c't v_0^{1/2}}{2}}$$

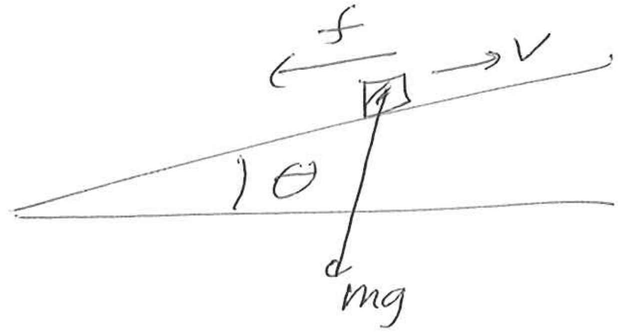
$$\boxed{\left(\frac{v}{v_0}\right) = \frac{1}{1 + c't v_0^{1/2}/2}}$$

$v(0) = v_0$

$v(t \to \infty) \to \dfrac{1}{\frac{c't v_0^{1/2}}{2}}$



2.27




$$m\dot{v} = -mg\sin\theta - cv^2$$

$$\dot{v} = -g\sin\theta - \bar{c}v^2; \qquad \bar{c} = \frac{(c/m)}{\cancel{g\sin\theta}}$$

Sep variables

$$\int \frac{dv}{g\sin\theta + \bar{c}v^2} = -\int dt$$

$$\int \frac{dv}{a^2+v^2} = -\bar{c}t + \text{Const}$$

$$a^2 \equiv \frac{g\sin\theta}{\bar{c}}$$

($a$ = term speed)

look up $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$$\Rightarrow \frac{1}{a}\tan^{-1}\left(\frac{v}{a}\right) = -\bar{c}t + \text{const}$$

$t=0$ $\quad \frac{1}{a}\tan^{-1}\left(\frac{v_0}{a}\right) = \text{const}$

$$\Rightarrow \boxed{\tan^{-1}(v/a) = \tan^{-1}(v_0/a) - a\bar{c}t}$$

Time to top ($v=0$) $\Rightarrow$ $\boxed{T = \frac{m}{ac}\tan^{-1}(v_0/a)}$