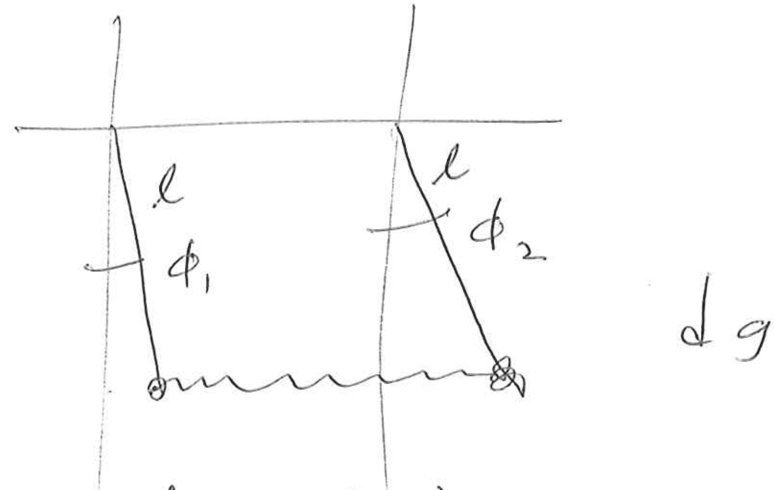
11.14



- $m_1 = m_2 = m$

$$T = \frac{1}{2} m \ell^2 (\dot\phi_1^2 + \dot\phi_2^2)$$

$$U_s = \frac{1}{2} k \ell^2 (\phi_2 - \phi_1)^2$$

$$U_g = \frac{1}{2} m g \ell (\phi_1^2 + \phi_2^2)$$

(A)

- let $m = 1$, $\ell = 1$, $k = 1$
  - $\Rightarrow$ length normalized to ℓ
  - $\Rightarrow$ time normalized to $(m/k)^{1/2}$
  - $\Rightarrow$ Energy normalized to $k\ell^2$

$$\Rightarrow \quad T = \frac{1}{2} (\dot\phi_1^2 + \dot\phi_2^2)$$

$$U_s = \frac{1}{2} (\phi_2 - \phi_1)^2$$

$$U_g = \frac{1}{2} g (\phi_1^2 + \phi_2^2)$$

(B)

- Note Cannot set $g = 1$.
  (a) g has same dimensions as $\ell k/m$
  (b) cannot set more than 3 things to unity.

- $\mathcal{L} = T - U_s - U_g$

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}_1} = \dot{\phi}_1 \; ; \; \frac{\partial \mathcal{L}}{\partial \phi_1} = (\phi_2 - \phi_1) - g\phi_1$$

$$\Rightarrow \boxed{\begin{aligned} \ddot{\phi}_1 &= -g\phi_1 + (\phi_2 - \phi_1) \\ \ddot{\phi}_2 &= -g\phi_2 - (\phi_2 - \phi_1) \end{aligned}}$$

System Depends only on one parameter g where

$$g \to \frac{mgl}{kl^2} = \frac{mg}{kl}$$

$$\Rightarrow \boxed{\begin{aligned} (g+1-\omega^2)\phi_1 &= \phi_2 \\ (g+1-\omega^2)\phi_2 &= \phi_1 \end{aligned}}$$

- $|Det| = 0 \Rightarrow (g+1-\omega^2)^2 = 1$

$$g+1-\omega^2 = \pm 1$$

#1 $\omega^2 = g \Rightarrow \phi_1 = \phi_2 \Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

#2 $\omega^2 = g+2 \Rightarrow \phi_1 = -\phi_2 \Rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

- More systematic normalization

- Start from (A)

let $\hat{U}_s = \dfrac{U_s}{kl^2}$, $\hat{U}_g = \dfrac{U_g}{kl^2}$, $\hat{T} = \dfrac{T}{kl^2}$

$$\Rightarrow \hat{T} = \frac{1}{2}\frac{m}{k}\left(\dot{\phi}_1^2 + \dot{\phi}_2^2\right)$$

$$\hat{U}_s = \frac{1}{2}\left(\phi_2 - \phi_1\right)^2$$

$$\hat{U}_g = \frac{1}{2}\frac{mg}{kl}\left(\phi_1^2 + \phi_2^2\right)$$

- let $\hat{t} = t\left(\dfrac{k}{m}\right)^{1/2}$

$$\Rightarrow \frac{d\phi_1}{dt} = \frac{d\hat{t}}{dt}\frac{d\phi_1}{d\hat{t}} = \left(\frac{k}{m}\right)^{1/2}\frac{d\phi_1}{d\hat{t}}$$

$$\Rightarrow \frac{1}{2}\frac{m}{k}\left(\frac{d\phi_1}{dt}\right)^2 = \frac{1}{2}\left(\frac{d\phi_1}{d\hat{t}}\right)^2$$

$$\Rightarrow \hat{T} = \frac{1}{2}\left(\hat{\dot{\phi}}_1^2 + \hat{\dot{\phi}}_2^2\right)$$

Drop all λ's

$\Rightarrow$

$$T = \frac{1}{2}\left(\dot{\phi}_1^2 + \dot{\phi}_2^2\right)$$

$$U_s = \frac{1}{2}\left(\phi_2 - \phi_1\right)^2$$

$$U_g = \frac{1}{2}\left(\frac{mg}{kl}\right)\left(\phi_1^2 + \phi_2^2\right)$$

— exactly as our starting point B

setting $k=1$, $l=1$, $m=1$,

with single param $g \to \frac{mg}{kl}$