**Phys410/F16/Hassam/Midterm 2**

**1 hr 40 min 1 sheet 2-sided Box all important results**

**Problem 1 (20 points)**

Find the equation of the path joining the origin (0,0) to the point (1,1) in the x-y plane that makes the integral ∫ dx (y’2 + y3y’ + y’2) an extremum, where y’=dy/dx.

**Problem 2 (20 points)**

A new physics equation is proposed to update Newton’s Equation. The new equation takes the form

m (d/dt)[x**.** + ε sinh(x**.**)] = F(x), where F(x) = - dU/dx, and x**.** = dx/dt.

If this equation is to be taken seriously, one must be able to recast it into standard Euler-Lagrange form, (d/dt)(∂L/∂x**.**) = (∂L/∂x), thus ensuring the existence of a variational principle. Here, L= L(x,x**.**). Lagrange was able to accomplish this for ε = 0 by introducing the idea that x could be treated as an independent variable thus allowing derivatives of the form (∂f/∂x**.**)x to replace any functions of x**.**. Here, in general, f = f(x,x**.**).

Using Lagrange’s approach, investigate if you can recast the above equation with constant ε into E-L form. If so, define the new L(x,x**.**). For partial credit, you may present the ε = 0 formulation.  **Problem 3 (30 points)**

A particle is constrained to move on the surface of a circular cone. The cone axis is the vertical z-axis, the vertex is at the origin (pointing down), and the half angle is α. There is gravity g pointing down. Mass = m.

1. Write down the Lagrangian, L, in terms of spherical polar coordinates (r,φ).

2. Find two equations of motion.

3. Interpretthe φ equation in terms of the angular momentum λ, and use it to find an ODE in terms of only r(t).

4. State how this equation makes sense if the angular momentum = 0.

5. Find a value for r, r0, in terms of λ, m, g, α, such that the particle can remain on a horizontal and circular path. What is the frequency of rotation in terms of r0 and the parameters?

6. Suppose the particle is given a small radial kick, ie, r(t) = r­0 + ε(t), where |ε| << r0. Is the circular motion stable to small perturbations? If so, what is the oscillation frequency normalized to the rotation frequency?

Useful: In spherical coordinates, d**r** = dr **r^** + r dθ **θ^** + r sinθ dφ **φ^**.

**Problem 4 (30 points)**

We want to study free fall and Coriolis deflection. We will use equations developed in Taylor’s text (Eqs 9.53 and Figure 9.15, attached). A mass m is released, from rest, from a height z=h, and x=y=0. The experiment is done at the equator (corresponding to

θ = π/2).

This problem has parameters Ω, g, and h. To simplify algebra, you may set g =1 and

h = 1, corresponding to distance normalized to h and time normalized to (h/g)1/2. You don’t have to do this, however.

1. Write down Eqs 9.53 as they apply at the Equator.

2. Using the initial conditions supplied, find y(t). Also, by performing an integration, find [dz/dt](t) in terms of x(t).

3. Insert the above results into the equation for d2x/dt2. Specify at least 3 characteristics of the resulting equation which point you to the methods to be used to solve this ODE.

4. Solve completely for x(t) given the boundary conditions. This will be an exact solution.

5. We now assume that the free-fall time from height h is given by the inertial frame result, ie, τ2 = 2h/g. From 4., find the deflection in x(t), d, as evaluated according to d = x(τ). Your answer should be a two term expression for d given in terms of Ω, g, h, and τ.

6. Identify in your expression for d the dimensionless parameter ε = Ωτ, proportional to hΩ2/g. Assuming ε << 1, make an approximation in d to get a nonzero expression for (d/h) which is proportional to εα. What is the exponent α?

Maybe useful: cos(x) = 1 - x2/2! + ….. sin(x) = x – x3/3! + …..