**Problem 13.1H** In this problem, we review some properties associated with Principal Axes of a rigid body RB). We first summarize the properties and then step thru the proofs.

**Summary**

For any RB, there exist three, orthogonal, Principal axes which have the following properties if physics is conducted w.r.t. to a coordinate system based on the P-axes: (1) the **I** tensor is diagonal; (2) if **ω** // P-axesthen **L** is also // **ω**, and vice-versa. Recall that **L** = **I.ω**.

To check all this, we step thru as below.

1. First, lets define P-axes as coordinate axes such that if **ω** is // to a P-axis, then **L // ω**. Assume also there is a theorem which says three P-axes always exist and P-axes are orthogonal (or can be made so).

2. As per the definition, to find the P-axes, we solve the equation **I.ω** =λ**ω** (i.e., L // **ω**)for all possible **ω**(n). We solve this in an arbitrary Cartesian system. If we can find such **ω**(n), we have the P-axes. The equation **I.ω** =λ**ω** is an eigenvalue equation. There is a theorem (related to the above and more precise) which says that an *n x n* real, symmetric matrix has *n* real eigenvalues, and *n* eigenvectors which are orthogonal. Since **I** is symmetric, we are assured we will find 3 P-axes, **ω**(n), and associated λ(n).

3. Now assume we have found three **ω**(n).  We now return to the RB and this time around we work in coordinates based on the P-axes. Note that in this coordinate system, the **ω**(n) are simply (1,0,0), (0,1,0), and (0,0,1). Also, **I** will look different. Let **I** be a general matrix with elements Ixx, Ixy, Ixz, etc, w.r.t. the P-axes. Since **I.ω**(n) = λ(n)**ω**(n), show, by considering each **ω**(n) in turn, that **I** must be diagonal, made up from the λ(n)’s, and that the λ(n)’s are just the moments of inertia corresponding to the respective axes.

4. Using the fact that **I** is diagonal, prove the property mentioned in the Summary, ie, **ω** // P-axes ⬄ **L** // **ω**. Your proof should work both ways. (The left to right proof is straightforward; the R to L one is not: make sure write down all 3 equations and solve them simultaneously.)