Phys410/F16/Hassam/Final Take Home
Out: Dec15 8am Due at my office: Dec15 by 3pm

Box all important results

1. You are expected to work independently. Please sign the below.

2. You may consult any material except consult with other persons.

3. Write your answers on any 8x11 worksheets. **STAPLE THE PAGES. DO NOT** submit more than 12 sheets altogether.

4. Show all but only essential work. You may use separate paper for scratch work: do not attach this.

5. **Box all answers in response to specific questions.**

6. Return exam to my office. There will be a box outside.

7. My office is in A V Williams, across from the Kim Bldg. 3d floor, Room 3307. AVW is a large horseshoe. As you approach from the Kim Bldg, use the entrance and stairs at the left tip of the horseshoe.

8. You may email me with questions. I will cc my replies to all if its of general interest.

I attest that that the work presented here is independent work. I have not consulted with other persons, live or online, in working this test.

Signature:__________________________

Name:__________________________

Date:__________________________

Problem 1 Modes (20 points)

Three oscillators of equal mass m are coupled so that the potential energy is $U = (k/2)[x_1^2 + x_2^2 + x_3^2 + \kappa(x_2x_3 + x_2x_1)]$. The kinetic energy is as usual, ie, $T = (m/2)[(dx_1/dt)^2 + (dx_2/dt)^2 + (dx_3/dt)^2]$. The coefficient $\kappa = \sqrt{2}$. Find the normal modes, both the frequencies and the eigenvectors. Is there a zero frequency mode? If so, what is the physics giving rise to this mode?

Problem 2 Diagonal (20 points)

A three particle system, rigidly connected, consists of equal masses m, m, m, each at Cartesian coordinates (x,y,z), as follows: (b,0,b), (b,b,-b), and (-b,b,0).

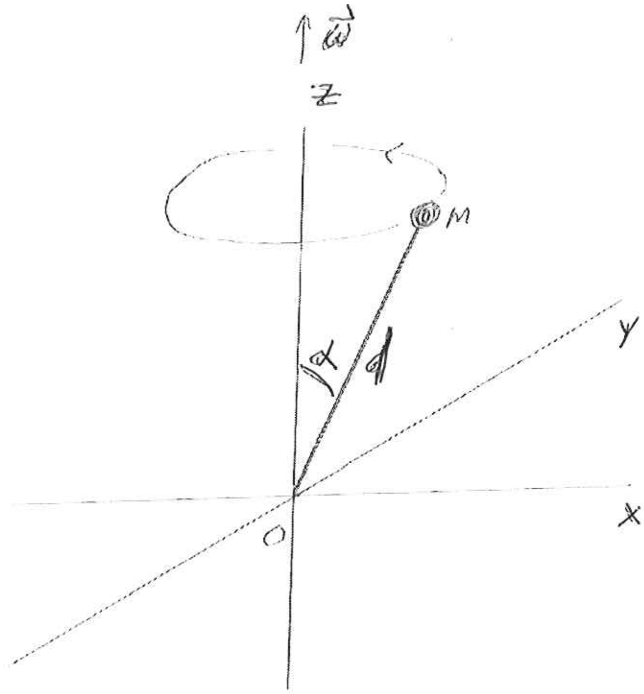
Find the inertia tensor, the principal axes, and the principal moments of inertia.

Problem 3 Special Rel (20 points)

A mass of rest mass m is thrown from the origin at t=0 at relativistic speed in the y-direction, with initial speed, v_0. [In general, the relativistic momentum is $\mathbf{p} = \gamma m\mathbf{v}$, where $\gamma^2 = 1/(1-\mathbf{v}.\mathbf{v}/c^2)$, and $\mathbf{v}(t)$ is the velocity]. The mass is subject to a constant force F_0 in the x-direction. Find the velocity of the mass, $\mathbf{v}(t) = [v_x(t), v_y(t)]$, as function of time. $\mathbf{v}(t)$ should be expressed only in terms of v_0, c, F_0/m, and t, or combinations thereof. You may set c=1 if you wish. Discuss the velocity for early times ($t \rightarrow 0$) and for very long times ($t \rightarrow \infty$).

Problem 4 Torque (40 points)

A mass m is connected to one end of a light rigid rod of length d. The other end of the rod is freely pivoted about the origin of a space-based Cartesian system (x,y,z). The system is *made to rotate* (by an external torque) such that the rotation frequency is $\omega = \omega_0 \hat{\mathbf{z}}$, where ω_0 is a constant, and the angle between the rod and the z-axis is fixed at α. See Figure.

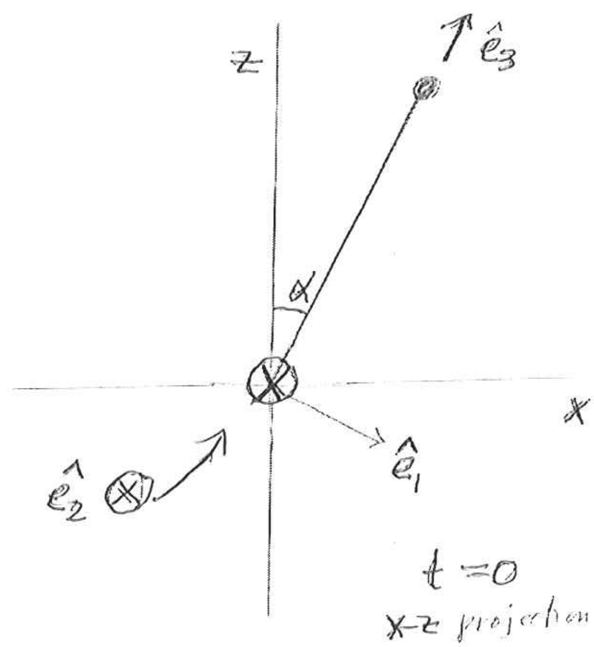


1. What differential equation is satisfied by the position vector of the mass, $\mathbf{r}(t) = [x(t), y(t), z(t)]$? Find $\mathbf{r}(t)$ for all t, w.r.t. the space-based coordinates, if, at t=0, y(0)=0.

In this problem, we are given $\omega(t)$ (ie, a constant) but we are not given the torque. However, since we know ω and $\mathbf{r}(t)$, we can calculate $d\mathbf{L}/dt$, and therefore find the required torque.

2. *Using only space-based coordinates*, and performing the calculation using an Inertia tensor, find the required torque $\Gamma(t)$. Your answer should use the unit vectors $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$.

We can now check our answer by doing this calculation in body-based coordinates. A snapshot of the system at t=0 is shown in the Figure. Three orthogonal body-based axes, $\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3$, are also shown. You are given that, as time goes on, the RB and the axes rotate in such a way that the vector $\hat{\mathbf{e}}_2$ always stays in the x-y plane. See Fig.



3. The 3 $\hat{\mathbf{e}}$ axes are principal axes. Confirm this by calculating the Inertia tensor in body coordinates and showing that it is diagonal.

4. Now perform a calculation *in body-based coordinates*, and *using equations appropriate for a diagonalized inertia tensor*, to find the required torque $\Gamma(t)$. Give your answer in the body-based $\hat{\mathbf{e}}$ vectors.

5. Compare your answer in 4. to that in 2. In particular, compare the torque directions (between parts 2 and 4) at t=0 and $t=\pi/(2\omega_0)$ and show that these are consistent w.r.t. the Figure above.