HOMEWORK#9 SOLUTION

[Chap28]

***1** •

Determine the Concept We know that the magnetic flux (in this case the magnetic field because the area of the conducting loop is constant and its orientation is fixed) must be changing so the only issues are whether the field is increasing or decreasing and in which direction. Because the direction of the magnetic field associated with the clockwise current is into the page, the changing field that is responsible for it must be either increasing out of the page (not included in the list of possible answers) or a decreasing field directed into the page. $\boxed{(d) \text{ is correct.}}$

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(*a*) False. The induced emf in a circuit is proportional to *the rate of change of* the magnetic flux through the circuit.

(*b*) True.

(*c*) True.

(*d*) False. The inductance of a solenoid is determined by its length, cross-sectional area, number of turns per unit length, and the permeability of the matter in its core.

***24** ••

Picture the Problem We can use the hint to set up the element of area dA and express the flux $d\phi_m$ through it and then carry out the details of the integration to express ϕ_m.

(*a*) Express the flux through the strip of area dA:

$$d\phi_m = BdA$$

where $dA = bdx$.

Express B at a distance x from a long, straight wire:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{x} = \frac{\mu_0}{2\pi}\frac{I}{x}$$

Substitute to obtain:

$$d\phi_m = \frac{\mu_0}{2\pi}\frac{I}{x}bdx = \frac{\mu_0 Ib}{2\pi}\frac{dx}{x}$$

Integrate from $x = d$ to $x = d + a$:

$$\phi_m = \frac{\mu_0 Ib}{2\pi}\int_d^{d+a}\frac{dx}{x} = \boxed{\frac{\mu_0 Ib}{2\pi}\ln\frac{d+a}{d}}$$

(*b*) Substitute numerical values and evaluate ϕ_m:

$$\phi_{\mathrm{m}} = \frac{\left(4\pi\times10^{-7}\ \mathrm{N/A^2}\right)(20\,\mathrm{A})(0.1\,\mathrm{m})}{2\pi}\ln\left(\frac{7\,\mathrm{cm}}{2\,\mathrm{cm}}\right) = \boxed{5.01\times10^{-7}\ \mathrm{Wb}}$$

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Picture the Problem We can find the induced emf by applying Faraday's law to the loop. The application of Ohm's law will yield the induced current in the loop and we can find the rate of joule heating using $P = I^2R$.

(*a*) Apply Faraday's law to express the induced emf in the loop in terms of the rate of change of the magnetic field:

$$|\mathcal{E}| = \frac{d\phi_{\mathrm{m}}}{dt} = \frac{d}{dt}(AB) = A\frac{dB}{dt} = \pi R^2\frac{dB}{dt}$$

Substitute numerical values and evaluate $|\mathcal{E}|$:

$$|\mathcal{E}| = \pi(0.05\,\mathrm{m})^2(40\,\mathrm{mT/s}) = \boxed{0.314\,\mathrm{mV}}$$

(*b*) Using Ohm's law, relate the induced current to the induced voltage and the resistance of the loop and evaluate *I*:

$$I = \frac{\mathcal{E}}{R} = \frac{0.314\,\mathrm{mV}}{0.4\,\Omega} = \boxed{0.785\,\mathrm{mA}}$$

(*c*) Express the rate at which power is dissipated in a conductor in terms of the induced current and the resistance of the loop and evaluate *P*:

$$P = I^2R = (0.785\,\mathrm{mA})^2(0.4\,\Omega)$$
$$= \boxed{0.247\ \mu\mathrm{W}}$$

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Picture the Problem Because the speed of the rod is constant, an external force must act on the rod to counter the magnetic force acting on the induced current. We can use the motional-emf equation $\mathcal{E} = vB\ell$ to evaluate the induced emf, Ohm's law to find the current in the circuit, Newton's 2nd law to find the force needed to move the rod with constant velocity, and $P = Fv$ to find the power input by the force.

(*a*) Relate the induced emf in the circuit to the speed of the rod, the magnetic field, and the length of the rod:

$$\mathcal{E} = vB\ell = (10\,\mathrm{m/s})(0.8\,\mathrm{T})(0.2\,\mathrm{m})$$
$$= \boxed{1.60\,\mathrm{V}}$$

(*b*) Using Ohm's law, relate the current in the circuit to the induced emf and the resistance of the circuit:

$$I = \frac{\mathcal{E}}{R} = \frac{1.6\,\text{V}}{2\,\Omega} = \boxed{0.800\,\text{A}}$$

Note that, because the rod is moving to the right, the flux in the region defined by the rod, the rails, and the resistor is increasing. Hence, in accord with Lenz's law, the current must be counterclockwise if its magnetic field is to counter this increase in flux.

(*c*) Because the rod is moving with constant velocity, the net force acting on it must be zero. Apply Newton's 2nd law to relate F to the magnetic force F_m:

$$\sum F_x = F - F_{\text{m}} = 0$$

and

$$F = F_{\text{m}} = BI\ell$$
$$= (0.8\,\text{T})(0.8\,\text{A})(0.2\,\text{m}) = \boxed{0.128\,\text{N}}$$

(*d*) Express the power input by the force in terms of the force and the velocity of the rod:

$$P = Fv = (0.128\,\text{N})(10\,\text{m/s}) = \boxed{1.28\,\text{W}}$$

(*e*) The rate of Joule heat production is given by:

$$P = I^2R = (0.8\,\text{A})^2(2\,\Omega) = \boxed{1.28\,\text{W}}$$

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Picture the Problem Let m be the mass of the rod and F be the net force acting on it due to the current in it. We can obtain the equation of motion of the rod by applying Newton's 2nd law to relate its acceleration to B, I, and ℓ. The net emf that drives I in this circuit is the emf of the battery minus the emf induced in the rod as a result of its motion.

(*a*) Letting the direction of motion of the rod be the positive x direction, apply $\sum F_x = ma_x$ to the rod:

$$BI\ell = m\frac{dv}{dt} \qquad (1)$$

where

$$I = \frac{\mathcal{E} - B\ell v}{R} \qquad (2)$$

Substitute to obtain:

$$\frac{dv}{dt} = \boxed{\frac{B\ell}{mR}(\mathcal{E} - B\ell v)}$$

(*b*) Express the condition on dv/dt when the rod has achieved its terminal speed:

$$\frac{B\ell}{mR}(\mathcal{E} - B\ell v_{\text{t}}) = 0$$

Solve for v_t to obtain:

$$v_t = \boxed{\frac{\mathcal{E}}{B\ell}}$$

(*c*) Substitute v_t for v in equation (2) to obtain:

$$I = \frac{\mathcal{E} - B\ell\dfrac{\mathcal{E}}{B\ell}}{R} = \boxed{0}$$

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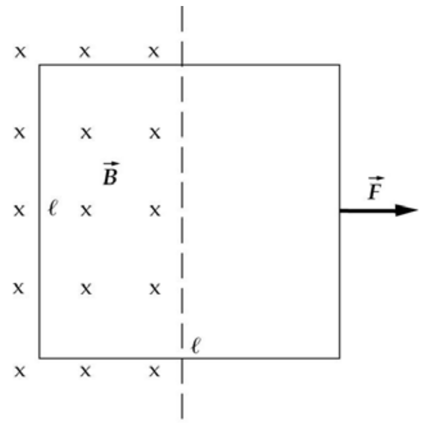
Picture the Problem The diagram shows the square loop being pulled from the magnetic field $\vec{B}$ by the constant force $\vec{F}$. The time required to pull the loop out of the magnetic field depends on the terminal speed of the loop. We can apply Newton's 2nd law and use the expressions for the magnetic force on a moving wire in a magnetic field to obtain the equation of motion for the loop and, from this equation, an expression for the terminal speed of the loop.



Apply $\sum \vec{F} = m\vec{a}$ to the square loop to obtain:

$$F - F_m = m\frac{dv}{dt} \quad (1)$$

The magnetic force is given by:

$$F_m = I\ell B = \frac{\mathcal{E}\ell B}{R}$$

where R is the resistance of the loop.

Substitute for Fm in equation (1) to obtain:

$$F - \frac{\mathcal{E}\ell B}{R} = m\frac{dv}{dt} \quad (2)$$

The induced emf ε is related to the speed of the loop:

$$\mathcal{E} = vB\ell$$

Substitute for in equation (2) to obtain the equation of motion of the loop:

$$F - \frac{\ell^2 B^2}{R}v = m\frac{dv}{dt}$$

When the loop reaches its terminal speed, $dv/dt = 0$ and:

$$F - \frac{\ell^2 B^2}{R}v_t = 0 \Rightarrow v_t = \frac{R}{\ell^2 B^2}F$$

This result tells us that doubling F doubles the terminal speed v_t. Hence, doubling F will halve the time required to pull the loop from the magnetic field and $\boxed{(c)\text{ is correct.}}$

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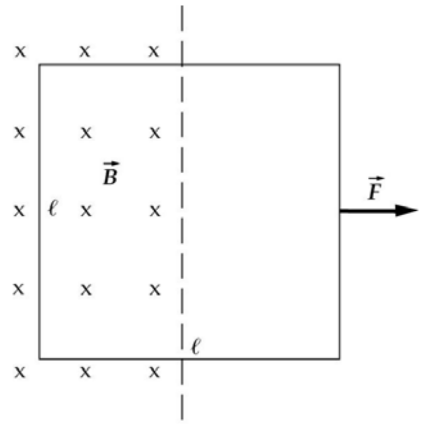

Apply $\sum \vec{F} = m\vec{a}$ to the square loop to obtain:

$$F - F_{\text{m}} = m\frac{dv}{dt} \qquad (1)$$

The magnetic force is given by:

$$F_{\text{m}} = I\ell B = \frac{\mathcal{E}\,\ell\,B}{R}$$

where R is the resistance of the loop.

Substitute for F_{m} in equation (1) to obtain:

$$F - \frac{\mathcal{E}\,\ell\,B}{R} = m\frac{dv}{dt} \qquad (2)$$

The induced emf ε is related to the speed of the loop:

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$$F - \frac{\ell^2 B^2}{R}v_{\text{t}} = 0 \Rightarrow v_{\text{t}} = \frac{R}{\ell^2 B^2}F$$

This result tells us that halving R halves the terminal speed v_{t}. Hence, halving R will double the time required to pull the loop from the magnetic field and $\boxed{(b)\text{ is correct.}}$

[Honors section]

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Picture the Problem Consider an element of area $dA = Ldr$ where $r \le R$. We can use its definition to express $d\phi_m$ through this area in terms of B and Ampere's law to express B as a function of I. The fact that the current is uniformly distributed over the cross-sectional area of the conductor allows us to set up a proportion from which we can obtain I as a function of r. With these substitutions in place we can integrate $d\phi_m$ to obtain ϕ_m/L.

Express the flux $d\phi_m$ through an area Ldr:

$$d\phi_m = BdA = BLdr \qquad (1)$$

Apply Ampere's law to the current contained inside a cylindrical region of radius $r < R$:

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell} = 2\pi rB = \mu_0 I_C$$

and

$$B = \frac{\mu_0 I_C}{2\pi r}$$

Using the fact that the current I is uniformly distributed over the cross-sectional area of the conductor, express its variation with distance r from the center of the conductor:

$$\frac{I(r)}{I} = \frac{\pi r^2}{\pi R^2}$$

or

$$I(r) = I_C = I\frac{r^2}{R^2}$$

Substitute and simplify to obtain:

$$B = \frac{\mu_0 I}{2\pi r}\frac{r^2}{R^2} = \frac{\mu_0 I}{2\pi R^2}r$$

Substitute in equation (1):

$$d\phi_m = \frac{\mu_0 LI}{2\pi R^2}rdr$$

Integrate $d\phi_m$ from $r = 0$ to $r = R$ to obtain:

$$\phi_m = \frac{\mu_0 LI}{2\pi R^2}\int_0^R rdr = \frac{\mu_0 LI}{4\pi}$$

Divide both sides of this equation by L to express the magnetic flux per unit length:

$$\frac{\phi_m}{L} = \boxed{\frac{\mu_0 I}{4\pi}}$$