[TA's Notice]

1. homework#7 [Chap25] #19

=> The solution was described incorrectly, and refer the position at t=0 in the figure 25-42

2. homework#8 [Chap27] #63,#67

=> Many students missed those problems. I graded that problem on some students' paper, but I canceled that grading. I do not understand why this kind of thing happened again.

HOMEWORK#8 SOLUTION

[Chap27]

***1** •

Picture the Problem The electric forces are described by Coulomb's law and the laws of attraction and repulsion of charges and are independent of the fact the charges are moving. The magnetic interaction is, on the other hand, dependent on the motion of the charges. Each moving charge constitutes a current that creates a magnet field at the location of the other charge.

(*a*) The electric forces are repulsive; the magnetic forces are attractive (the two charges moving in the same direction act like two currents in the same direction).

(*b*) The electric forces are again repulsive; the magnetic forces are also repulsive.

4 •

Determine the Concept Applying the right-hand rule to the wire to the left we see that the magnetic field due to its current is out of the page at the midpoint. Applying the right-hand rule to the wire to the right we see that the magnetic field due to its current is out of the page at the midpoint. Hence, the sum of the magnetic fields is out of the page as well. $\boxed{(c)\text{ is correct.}}$

5 •

Determine the Concept While we could express the force wire 1 exerts on wire 2 and compare it to the force wire 2 exerts on wire 1 to show that they are the same, it is simpler to recognize that these are action and reaction forces. $\boxed{(a)\text{ is correct.}}$

***18** •

Determine the Concept The force per unit length experienced by each segment of the wire, due to the currents in the other segments of the wire, will be equal. These equal forces will result in the wire tending to form a circle.

25 •

Picture the Problem We can substitute for $\vec{v}$ and q in the equation describing the magnetic field of the moving proton ($\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for each of the given points of interest, and substitute to find $\vec{B}$.

The magnetic field of the moving proton is given by:

$$\vec{B}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}=\left(10^{-7}\ \text{N/A}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)\frac{\left[\left(10^4\ \text{m/s}\right)\hat{i}+\left(2\times10^4\ \text{m/s}\right)\right]\hat{j}\times\hat{r}}{r^2}$$

$$=\left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{i}+2\hat{j}\right)\times\hat{r}}{r^2}$$

(*a*) Find r and $\hat{r}$ for the proton at (3 m, 4 m) and the point of interest at (2 m, 2 m):

$\vec{r}=-(1\,\text{m})\hat{i}-(2\,\text{m})\hat{j}$, $r=\sqrt{5}\,\text{m}$, and

$$\hat{r}=-\frac{1}{\sqrt{5}}\hat{i}-\frac{2}{\sqrt{5}}\hat{j}$$

Substitute for $\hat{r}$ and evaluate $\vec{B}(1\,\text{m},3\,\text{m})$:

$$\vec{B}(1\,\text{m},3\,\text{m})=\left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{i}+2\hat{j}\right)\times\left(-\frac{1}{\sqrt{5}}\hat{i}-\frac{2}{\sqrt{5}}\hat{j}\right)}{r^2}$$

$$=\frac{\left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)}{\sqrt{5}}\left[\frac{-2\hat{k}+2\hat{k}}{\left(\sqrt{5}\,\text{m}\right)^2}\right]=\boxed{0}$$

(*b*) Find r and $\hat{r}$ for the proton at (3 m, 2 m) and the point of interest at (6 m, 4 m):

$\vec{r}=(3\,\text{m})\hat{i}$, $r=3\,\text{m}$, and $\hat{r}=\hat{i}$

Substitute for $\hat{r}$ and evaluate $\vec{B}(6\,\text{m},4\,\text{m})$:

$$\vec{B}(6\,\text{m},4\,\text{m})=\left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{i}+2\hat{j}\right)\times\hat{i}}{(3\,\text{m})^2}=\left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\left(\frac{-2\hat{k}}{9\,\text{m}^2}\right)$$

$$=\boxed{-\left(3.56\times10^{-23}\ \text{T}\right)\hat{k}}$$

(*c*) Find r and $\hat{r}$ for the proton at (3 m, 4 m) and the point of interest at the (3 m, 6 m):

$\vec{r}=(2\,\text{m})\hat{j}$, $r=2\,\text{m}$, and $\hat{r}=\hat{j}$

Substitute for $\hat{r}$ and evaluate $\vec{B}(3\,\text{m},6\,\text{m})$:

$$\vec{B}(3\,\text{m},6\,\text{m})=\left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{i}+2\hat{j}\right)\times\hat{j}}{(2\,\text{m})^2}=\left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\left(\frac{\hat{k}}{4\,\text{m}^2}\right)$$

$$=\boxed{\left(4.00\times10^{-23}\ \text{T}\right)\hat{k}}$$

26 •

Picture the Problem The centripetal force acting on the orbiting electron is the Coulomb force between the electron and the proton. We can apply Newton's 2nd law to the electron to find its orbital speed and then use the expression for the magnetic field of a moving charge to find *B*.

Express the magnetic field due to the motion of the electron:

$$B = \frac{\mu_0}{4\pi}\frac{ev}{r^2}$$

Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to the electron:

$$\frac{ke^2}{r^2} = m\frac{v^2}{r}$$

Solve for v to obtain:

$$v = \sqrt{\frac{ke^2}{mr}}$$

Substitute and simplify to obtain:

$$B = \frac{\mu_0}{4\pi}\frac{e}{r^2}\sqrt{\frac{ke^2}{mr}} = \frac{\mu_0 e^2}{4\pi r^2}\sqrt{\frac{k}{mr}}$$

Substitute numerical values and evaluate *B*:

$$B = \frac{\left(10^{-7}\ \text{N/A}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{\left(5.29\times10^{-11}\ \text{m}\right)^2}\sqrt{\frac{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.29\times10^{-11}\ \text{m}\right)}} = \boxed{12.5\,\text{T}}$$

***27** ••

Picture the Problem We can find the ratio of the magnitudes of the magnetic and electrostatic forces by using the expression for the magnetic field of a moving charge and Coulomb's law. Note that v and $\vec{r}$, where $\vec{r}$ is the vector from one charge to the other, are at right angles. The field $\vec{B}$ due to the charge at the origin at the location (0, *b*, 0) is perpendicular to *v* and $\vec{r}$.

Express the magnitude of the magnetic force on the moving charge at (0, *b*, 0):

$$F_B = qvB = \frac{\mu_0}{4\pi}\frac{q^2v^2}{b^2}$$

and, applying the right hand rule, we find that the direction of the force is toward the charge at the origin; i.e., the magnetic force between the two moving charges is attractive.

Express the magnitude of the repulsive electrostatic interaction between the two charges:

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{b^2}$$

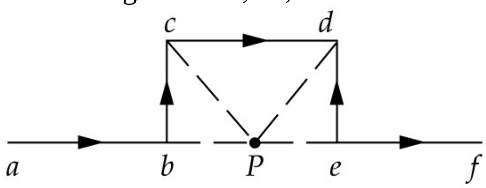
Express the ratio of F_B to F_E and simplify to obtain:

$$\frac{F_B}{F_E} = \frac{\frac{\mu_0}{4\pi}\frac{q^2v^2}{b^2}}{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{b^2}} = \varepsilon_0\mu_0 v^2 = \boxed{\frac{v^2}{c^2}}$$

where c is the speed of light in a vacuum.

50 ••

Picture the Problem Note that the current segments *a-b* and *e-f* do not contribute to the magnetic field at point *P*. The current in the segments *b-c*, *c-d*, and *d-e* result in a magnetic field at *P* that points into the plane of the paper. Note that the angles *bPc* and *ePd* are 45° and use the expression for *B* due to a straight wire segment to find the contributions to the field at P of segments *bc*, *cd*, and *de*.



Express the resultant magnetic field at *P*:

$$B = B_{bc} + B_{cd} + B_{de}$$

Express the magnetic field due to a straight line segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right) \qquad (1)$$

Use equation (1) to express B_{bc} and B_{de}:

$$B_{bc} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin 45° + \sin 0°\right) = \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Use equation (1) to express B_{cd}:

$$B_{cd} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin 45° + \sin 45°\right) = 2\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Substitute to obtain:

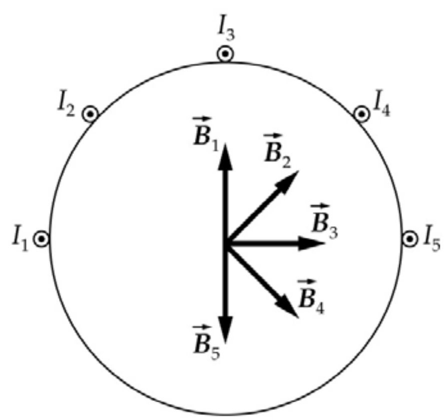
$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° + 2\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° + \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° = 4\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Substitute numerical values and evaluate B:

$$B = 4\left(10^{-7}\ \mathrm{T\cdot m/A}\right)\frac{8\,\mathrm{A}}{0.01\,\mathrm{m}}\sin 45° = \boxed{226\,\mu\mathrm{T}}$$

58 ••

Picture the Problem The configuration is shown in the adjacent figure. Here the z axis points out of the plane of the paper, the x axis points to the right, the y axis points up. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule to find the magnetic field due to the current in each wire and add these magnetic fields vectorially to find the resultant field.



Express the resultant magnetic field on the z axis:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5$$

$\vec{B}_1$ is given by:

$$\vec{B}_1 = B\hat{j}$$

$\vec{B}_2$ is given by:

$$\vec{B}_2 = (B\cos 45°)\hat{i} + (B\sin 45°)\hat{j}$$

$\vec{B}_3$ is given by:

$$\vec{B}_3 = B\hat{i}$$

$\vec{B}_4$ is given by:

$$\vec{B}_4 = (B\cos 45°)\hat{i} - (B\sin 45°)\hat{j}$$

$\vec{B}_5$ is given by:

$$\vec{B}_5 = -B\hat{j}$$

Substitute for $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, $\vec{B}_4$, and $\vec{B}_5$ and simplify to obtain:

$$\begin{aligned}\vec{B} &= B\hat{j} + (B\cos 45°)\hat{i} + (B\sin 45°)\hat{j} + B\hat{i} + (B\cos 45°)\hat{i} - (B\sin 45°)\hat{j} - B\hat{j} \\ &= (B\cos 45°)\hat{i} + B\hat{i} + (B\cos 45°)\hat{i} = (B + 2B\cos 45°)\hat{i} = \left(1+\sqrt{2}\right)B\hat{i}\end{aligned}$$

Express B due to each current at $z = 0$:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

Substitute to obtain:

$$\vec{B} = \boxed{\left(1+\sqrt{2}\right)\frac{\mu_0 I}{2\pi R}\hat{i}}$$

***63 •**

Picture the Problem We can apply Ampère's law to a circle centered on the axis of the cylinder and evaluate this expression for $r < R$ and $r > R$ to find B inside and outside the cylinder.

Apply Ampère's law to a circle centered on the axis of the cylinder:

$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I_C$$

Note that, by symmetry, the field is the same everywhere on this circle.

Evaluate this expression for $r < R$:

$$\oint_C \vec{B}_{\text{inside}}\cdot d\vec{\ell} = \mu_0(0) = 0$$

Solve for B_{inside} to obtain:

$$B_{\text{inside}} = \boxed{0}$$

Evaluate this expression for $r > R$:

$$\oint_C \vec{B}_{\text{outside}}\cdot d\vec{\ell} = B(2\pi R) = \mu_0 I$$

Solve for B_{outside} to obtain:

$$B_{\text{outside}} = \boxed{\frac{\mu_0 I}{2\pi R}}$$

69 ••

Picture the Problem We can use Ampère's law to calculate B because of the high degree of symmetry. The current through C depends on whether r is less than or the inner radius a, greater than the inner radius a but less than the outer radius b, or greater than the outer radius b.

(*a*) Apply Ampère's law to a circular path of radius $r < a$ to obtain:

$$\oint_C \vec{B}_{r<a}\cdot d\vec{\ell} = \mu_0 I_C = \mu_0(0) = 0$$

and

$$B_{r<a} = \boxed{0}$$

(*b*) Use the uniformity of the current over the cross-section of the conductor to express the current I' enclosed by a circular path whose radius satisfies the condition $a < r < b$:

$$\frac{I'}{\pi\left(r^2 - a^2\right)} = \frac{I}{\pi\left(b^2 - a^2\right)}$$

Solve for $I_C = I'$:

$$I_C = I' = I\,\frac{r^2 - a^2}{b^2 - a^2}$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{\boldsymbol{B}}_{a<r<b} \cdot d\vec{\ell} = B_{a<r<b}\left(2\pi r\right) = \mu_0 I' = \mu_0 I\,\frac{r^2 - a^2}{b^2 - a^2}$$

Solve for $B_{a<r<b}$:

$$B_{a<r<b} = \boxed{\frac{\mu_0 I}{2\pi r}\,\frac{r^2 - a^2}{b^2 - a^2}}$$

(*c*) Express I_C for $r > b$:

$$I_C = I$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{\boldsymbol{B}}_{r>b} \cdot d\vec{\ell} = B_{r>b}\left(2\pi r\right) = \mu_0 I$$

Solve for $B_{r>b}$:

$$B_{r>b} = \boxed{\frac{\mu_0 I}{2\pi r}}$$

100 •

Picture the Problem Because point *P* is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at *P*. Hence, we can use the expression for the magnetic field at the center of a current loop to find B_P.

Express the magnetic field at the center of a current loop:

$$B = \frac{\mu_0 I}{2R}$$

where *R* is the radius of the loop.

Express the magnetic field at the center of half a current loop:

$$B = \frac{1}{2}\,\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$$

Substitute numerical values and evaluate B:

$$B = \frac{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(15\,\text{A}\right)}{4\left(0.2\,\text{m}\right)} = \boxed{2.36 \times 10^{-5}\ \text{T}}$$

103 ••

Picture the Problem The magnetic field at P (which is out of the page) is the sum of the magnetic fields due to the three parts of the wire. Let the numerals 1, 2, and 3 denote the left-hand, center (short), and right-hand wires. We can then use the expression for B due to a straight wire segment to find each of these fields and their sum.

Express the resultant magnetic field at point P:

$$B_P = B_1 + B_2 + B_3$$

Because $B_1 = B_3$:

$$B_P = 2B_1 + B_2$$

Express the magnetic field due to a straight wire segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right)$$

For wires 1 and 3 (the long wires), $\theta_1 = 90°$ and $\theta_2 = 45°$:

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\sin 90° + \sin 45°\right) = \frac{\mu_0}{4\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}}\right)$$

For wire 2, $\theta_1 = \theta_2 = 45°$:

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\sin 45° + \sin 45°\right) = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{2}{\sqrt{2}}\right)$$

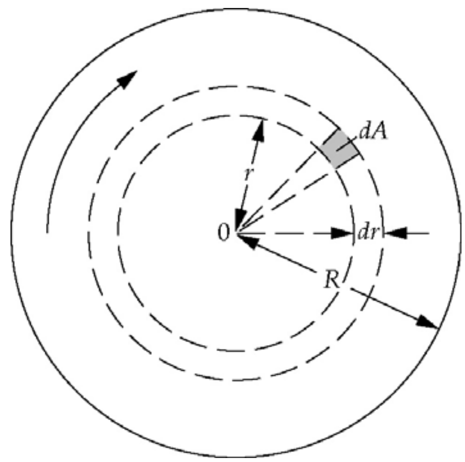
Substitute and simplify to obtain:

$$B_P = 2\left[\frac{\mu_0}{4\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}}\right)\right] + \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{2}{\sqrt{2}}\right) = \frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \frac{2}{\sqrt{2}}\right) = \boxed{\frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \sqrt{2}\right)}$$

[Honors section: chap27]

***125** •••

Picture the Problem The diagram shows the rotating disk and the circular strip of radius r and width dr with charge dq. We can use the definition of surface charge density to express dq in terms of r and dr and the definition of current to show that $dI = \omega\sigma r\,dr$. We can then use this current and expression for the magnetic field on the axis of a current loop to obtain the results called for in (*b*) and (*c*).



(*a*) Express the total charge dq that passes a given point on the circular strip once each period:

$$dq = \sigma dA = 2\pi\sigma r dr$$

Using its definition, express the current in the element of width dr:

$$dI = \frac{dq}{dt} = \frac{2\pi\sigma r dr}{\frac{2\pi}{\omega}} = \boxed{\omega\sigma r dr}$$

(*c*) Express the magnetic field dB_x at a distance x along the axis of the disk due to the current loop of radius r and width dr:

$$dB_x = \frac{\mu_0}{4\pi}\frac{2\pi r^2 dI}{\left(x^2+r^2\right)^{3/2}} = \frac{\mu_0\omega\sigma r^3}{2\left(x^2+r^2\right)^{3/2}}dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$B_x = \frac{\mu_0\omega\sigma}{2}\int_0^R \frac{r^3}{\left(x^2+r^2\right)^{3/2}}dr = \boxed{\frac{\mu_0\omega\sigma}{2}\left(\frac{R^2+2x^2}{\sqrt{R^2+x^2}}-2x\right)}$$

(*b*) Evaluate B_x for $x = 0$:

$$B_x(0) = \frac{\mu_0\omega\sigma}{2}\left(\frac{R^2}{\sqrt{R^2}}\right) = \boxed{\tfrac{1}{2}\mu_0\sigma\omega R}$$

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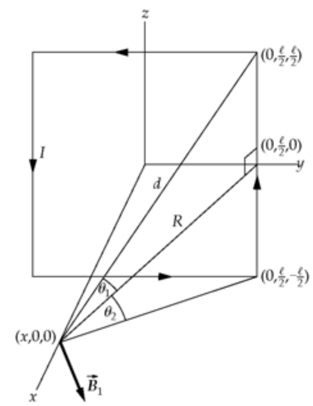
Picture the Problem From the symmetry of the system it is evident that the fields due to each segment of length ℓ are the same in magnitude. We can express the magnetic field at (x,0,0) due to one side (segment) of the square, find its component in the x direction, and then multiply by four to find the resultant field.



Express B due to a straight wire segment:

$$B=\frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1+\sin\theta_2\right)$$

where R is the perpendicular distance from the wire segment to the field point.

Use $\theta_1 = \theta_2$ and $R=\sqrt{x^2+\ell^2/4}$ to express B due to one side at (x,0,0):

$$B_1(x,0,0)=\frac{\mu_0}{4\pi}\frac{I}{\sqrt{x^2+\frac{\ell^2}{4}}}\left(2\sin\theta_1\right)$$

$$=\frac{\mu_0}{2\pi}\frac{I}{\sqrt{x^2+\frac{\ell^2}{4}}}\left(\sin\theta_1\right)$$

Referring to the diagram, express $\sin\theta_1$:

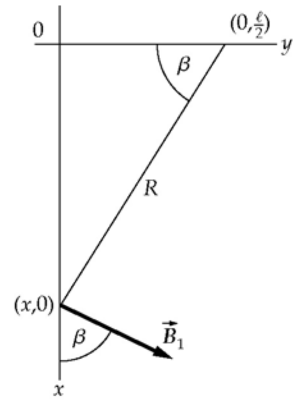
$$\sin\theta_1=\frac{\frac{\ell}{2}}{d}=\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{2}}}$$

Substitute to obtain:

$$B_1(x,0,0)=\frac{\mu_0}{2\pi}\frac{I}{\sqrt{x^2+\frac{\ell^2}{4}}}\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{2}}}$$

$$=\frac{\mu_0 I}{4\pi\sqrt{x^2+\frac{\ell^2}{4}}}\frac{\ell}{\sqrt{x^2+\frac{\ell^2}{2}}}$$

By symmetry, the sum of the y and z components of the fields due to the four segments must vanish, whereas the x components will add. The diagram to the right is a view of the xy plane showing the relationship between $\vec{B}_1$ and the angle β it makes with the x axis.



Express B_{1x}:

$$B_{1x} = B_1 \cos\beta$$

Substitute and simplify to obtain:

$$B_{1x} = \frac{\mu_0 I}{4\pi\sqrt{x^2+\frac{\ell^2}{4}}}\frac{\ell}{\sqrt{x^2+\frac{\ell^2}{2}}}\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{4}}}$$

$$= \frac{\mu_0 I\ell^2}{8\pi\left(x^2+\frac{\ell^2}{4}\right)\sqrt{x^2+\frac{\ell^2}{2}}}$$

The resultant magnetic field is the sum of the fields due to the 4 wire segments (sides of the square):

$$\vec{B} = 4B_{1x}\hat{\boldsymbol{i}}$$

$$= \boxed{\frac{\mu_0 I\ell^2}{2\pi\left(x^2+\frac{\ell^2}{4}\right)\sqrt{x^2+\frac{\ell^2}{2}}}\hat{\boldsymbol{i}}}$$

Factor x^2 from the two factors in the denominator to obtain:

$$\vec{B} = \frac{\mu_0 I\ell^2}{2\pi x^2\left(1+\frac{\ell^2}{4x^2}\right)\sqrt{x^2\left(1+\frac{\ell^2}{2x^2}\right)}}\hat{\boldsymbol{i}}$$

$$= \frac{\mu_0 I\ell^2}{2\pi x^3\left(1+\frac{\ell^2}{4x^2}\right)\sqrt{\left(1+\frac{\ell^2}{2x^2}\right)}}\hat{\boldsymbol{i}}$$

For $x >> \ell$:

$$\vec{B} \approx \frac{\mu_0 I\ell^2}{2\pi x^3}\hat{\boldsymbol{i}} = \boxed{\frac{\mu_0 \mu}{2\pi x^3}\hat{\boldsymbol{i}}}$$

where $\mu = I\ell^2$.