HOMEWORK#6 SOLUTION

[Chap 24]

***31 ••**

Picture the Problem We can relate the charge Q on the positive plate of the capacitor to the charge density of the plate σ using its definition. The charge density, in turn, is related to the electric field between the plates according to $\sigma = \epsilon_0 E$ and the electric field can be found from $E = \Delta V/\Delta d$. We can use $\Delta U = \frac{1}{2}Q\Delta V$ in part (*b*) to find the increase in the energy stored due to the movement of the plates.

(*a*) Express the charge Q on the positive plate of the capacitor in terms of the plate's charge density σ and surface area A:

$$Q = \sigma A$$

Relate σ to the electric field E between the plates of the capacitor:

$$\sigma = \epsilon_0 E$$

Express E in terms of the change in V as the plates are separated a distance Δd:

$$E = \frac{\Delta V}{\Delta d}$$

Substitute for σ and E to obtain:

$$Q = \epsilon_0 EA = \epsilon_0 A\frac{\Delta V}{\Delta d}$$

Substitute numerical values and evaluate Q:

$$Q = \left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(500\ \mathrm{cm^2}\right)\frac{100\ \mathrm{V}}{0.4\ \mathrm{cm}} = \boxed{11.1\ \mathrm{nC}}$$

(*b*) Express the change in the electrostatic energy in terms of the change in the potential difference:

$$\Delta U = \tfrac{1}{2}Q\Delta V$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = \tfrac{1}{2}(11.1\ \mathrm{nC})(100\ \mathrm{V}) = \boxed{0.553\ \mu\mathrm{J}}$$

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Picture the Problem Let C_1 represent the capacitance of the 1.2-μF capacitor and C_2 the capacitance of the 2^{nd} capacitor. Note that when they are connected as described in the problem statement they are in parallel and, hence, share a common potential difference. We can use the equation for the equivalent capacitance of two capacitors in parallel and the definition of capacitance to relate C_2 to C_1 and to the charge stored in and the potential difference across the equivalent capacitor. In part (*b*) we can use $U = \frac{1}{2}CV^2$ to find the energy before and after the connection was made and, hence, the energy lost when the connection was made.

(*a*) Using the definition of capacitance, find the charge on capacitor C_1:

$$Q_1 = C_1V = (1.2\,\mu\text{F})(30\,\text{V}) = 36\,\mu\text{C}$$

Express the equivalent capacitance of the two-capacitor system and solve for C_2:

$$C_{eq} = C_1 + C_2$$

and

$$C_2 = C_{eq} - C_1$$

Using the definition of capacitance, express C_{eq} in terms of Q_2 and V_2:

$$C_{eq} = \frac{Q_2}{V_2} = \frac{Q_1}{V_2}$$

where V_2 is the common potential difference (they are in parallel) across the two capacitors.

Substitute to obtain:

$$C_2 = \frac{Q_1}{V_2} - C_1$$

Substitute numerical values and evaluate C_2:

$$C_2 = \frac{36\,\mu\text{C}}{10\,\text{V}} - 1.2\,\mu\text{F} = \boxed{2.40\,\mu\text{F}}$$

(*b*) Express the energy lost when the connections are made in terms of the energy stored in the capacitors before and after their connection:

$$\begin{aligned}\Delta U &= U_{\text{before}} - U_{\text{after}} \\ &= \tfrac{1}{2}C_1V_1^2 - \tfrac{1}{2}C_{eq}V_f^2 \\ &= \tfrac{1}{2}\left(C_1V_1^2 - C_{eq}V_f^2\right)\end{aligned}$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = \tfrac{1}{2}\left[(1.2\,\mu\text{F})(30\,\text{V})^2 - (3.6\,\mu\text{F})(10\,\text{V})^2\right] = \boxed{360\,\mu\text{J}}$$

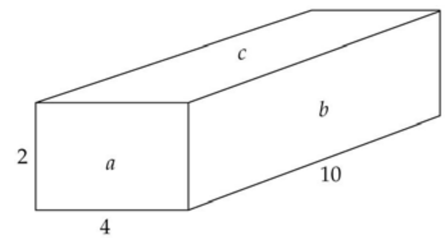
[Chap 25]

***1** •

Determine the Concept When current flows, the charges are not in equilibrium. In that case, the electric field provides the force needed for the charge flow.

***5** ••

Picture the Problem The resistance of the metal bar varies directly with its length and inversely with its cross-sectional area. Hence, to minimize the resistance of the bar, we should connect to the surface for which the ratio of the length to the contact area is least.



Denoting the surfaces as *a*, *b*, and *c*, complete the table to the right:

Surface	L	A	L/A
a	10	8	0.8
b	4	20	0.2
c	2	40	0.05

Because connecting to surface *c* minimizes *R*:

$$\boxed{(c)\text{ is correct.}}$$

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Picture the Problem The power dissipated in the resistor is given by $P = I^2R$. We can express the power dissipated when the current is $3I$ and, assuming that the resistance does not change, express the ratio of the two rates of energy dissipation to find the power dissipated when the current is $3I$.

Express the power dissipated in the resistor when the current in it is I:

$$P = I^2R$$

Express the power dissipated in the resistor when the current in it is $3I$:

$$P' = (3I)^2R = 9I^2R$$

Divide the second of these equations by the first to obtain:

$$\frac{P'}{P} = \frac{9I^2R}{I^2R} = 9$$

or

$$P' = 9P \text{ and } \boxed{(d)\text{ is correct.}}$$

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Picture the Problem Because the potential difference across the two combinations of resistors is constant, we can use $P = V^2/R$ to relate the power delivered by the battery to the equivalent resistance of each combination of resistors.

Express the power delivered by the battery when the resistors are connected in series:

$$P_s = \frac{V^2}{R_{eq}}$$

Letting R represent the resistance of the identical resistors, express R_{eq}:

$$R_{eq} = R + R = 2R$$

Substitute to obtain:

$$P_s = \frac{V^2}{2R} \qquad (1)$$

Express the power delivered by the battery when the resistors are connected in parallel:

$$P_p = \frac{V^2}{R_{eq}}$$

Express the equivalent resistance of the identical resistors connected in parallel:

$$R_{eq} = \frac{(R)(R)}{R + R} = \tfrac{1}{2}R$$

Substitute to obtain:

$$P_p = \frac{V^2}{\frac{1}{2}R} = \frac{2V^2}{R} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{P_p}{P_s} = \frac{\frac{2V^2}{R}}{\frac{V^2}{2R}} = 4$$

Solve for and evaluate P_p:

$$P_p = 4P_s = 4(20\text{ W}) = 80\text{ W}$$

and $\boxed{(e)\text{ is correct.}}$

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Picture the Problem We can solve $K = \frac{1}{2}m_e v^2$ for the velocity of an electron in the beam and use the relationship between current and drift velocity to find the beam current.

(a) Express the kinetic energy of the beam:

$$K = \tfrac{1}{2}m_e v^2$$

Solve for v:

$$v = \sqrt{\frac{2K}{m_e}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(10\text{ keV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = \boxed{5.93\times10^{7}\text{ m/s}}$$

(b) Use the relationship between current and drift velocity (here the velocity of an electron in the beam) to obtain:

$$I = nev_d A$$

Express the cross-sectional area of the beam in terms of its diameter D:

$$A = \tfrac{1}{4}\pi D^2$$

Substitute to obtain:

$$I = \tfrac{1}{4}\pi nev_d D^2$$

Substitute numerical values and evaluate I:

Substitute numerical values and evaluate I:

$$I = \tfrac{1}{4}\pi\left(5\times10^{6}\text{ cm}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(5.93\times10^{7}\text{ m/s}\right)\left(10^{-3}\text{ m}\right)^2 = \boxed{37.3\,\mu\text{A}}$$

***97** ••

Picture the Problem Let I_1 be the current delivered by the left battery, I_2 the current delivered by the right battery, and I_3 the current through the 6-Ω resistor, directed down. We can apply Kirchhoff's rules to obtain three equations that we can solve simultaneously for I_1, I_2, and I_3. Knowing the currents in each branch, we can use Ohm's law to find the potential difference between points a and b and the power delivered by both the sources.

(a) Apply Kirchhoff's junction rule at junction a:

$$I_1 + I_2 = I_3$$

Apply Kirchhoff's loop rule to a loop around the outside of the circuit to obtain:

$$12\text{ V} - (4\,\Omega)I_1 + (3\,\Omega)I_2 - 12\text{ V} = 0$$

or

$$-(4\,\Omega)I_1 + (3\,\Omega)I_2 = 0$$

Apply Kirchhoff's loop rule to a

$$12\text{ V} - (4\,\Omega)I_1 - (6\,\Omega)I_3 = 0$$

loop around the left-hand branch of the circuit to obtain:

Solve these equations simultaneously to obtain:

$$I_1 = \boxed{0.667\,\text{A}}\,,$$
$$I_2 = \boxed{0.889\,\text{A}}\,,$$
and
$$I_3 = \boxed{1.56\,\text{A}}$$

(*b*) Apply Ohm's law to find the potential difference between points *a* and *b*:

$$V_{ab} = (6\,\Omega)I_3 = (6\,\Omega)(1.56\,\text{A}) = \boxed{9.36\,\text{V}}$$

(*c*) Express the power delivered by the 12-V battery in the left-hand branch of the circuit:

$$P_{\text{left}} = \mathcal{E}I_1 = (12\,\text{V})(0.667\,\text{A}) = \boxed{8.00\,\text{W}}$$

Express the power delivered by the 12-V battery in the right-hand branch of the circuit:

$$P_{\text{right}} = \mathcal{E}I_2 = (12\,\text{V})(0.889\,\text{A}) = \boxed{10.7\,\text{W}}$$

[Honors section: chap24]

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Picture the Problem Note that, with switch S closed, C_1 and C_2 are in parallel and we can use $U_{closed} = \frac{1}{2}C_{eq}V^2$ and $C_{eq} = C_1 + C_2$ to obtain an equation we can solve for C_2. We can use the definition of capacitance to express Q_2 in terms of V_2 and C_2 and $U_{open} = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$ to obtain an equation from which we can determine V_2.

Express the energy stored in the capacitors after the switch is closed:

$$U_{closed} = \tfrac{1}{2}C_{eq}V^2$$

Express the equivalent capacitance of C_1 and C_2 in parallel:

$$C_{eq} = C_1 + C_2$$

Substitute to obtain:

$$U_{closed} = \tfrac{1}{2}(C_1 + C_2)V^2$$

Solve for C_2:

$$C_2 = \frac{2U_{closed}}{V^2} - C_1$$

Substitute numerical values and evaluate C_2:

$$C_2 = \frac{2(960\ \mu\text{J})}{(80\ \text{V})^2} - 0.2\ \mu\text{F} = \boxed{0.100\ \mu\text{F}}$$

Express the charge on C_2 when the switch is open:

$$Q_2 = C_2V_2 \quad (1)$$

Express the energy stored in the capacitors with the switch open:

$$U_{open} = \tfrac{1}{2}C_1V_1^2 + \tfrac{1}{2}C_2V_2^2$$

Solve for V_2 to obtain:

$$V_2 = \sqrt{\frac{2U_{open} - C_1V_1^2}{C_2}}$$

Substitute in equation (1) to obtain:

$$Q_2 = C_2\sqrt{\frac{2U_{\text{open}} - C_1V_1^2}{C_2}} = \sqrt{C_2\left(2U_{\text{open}} - C_1V_1^2\right)}$$

Substitute numerical values and evaluate Q_2:

$$Q_2 = \sqrt{(0.1\,\mu\text{F})\left[2(1440\,\mu\text{J}) - (0.2\,\mu\text{F})(40\,\text{V})^2\right]} = \boxed{16.0\,\mu\text{C}}$$