HOMEWORK#5 SOLUTION

***1 •**

Determine the Concept The capacitance of a parallel-plate capacitor is a function of the surface area of its plates, the separation of these plates, and the electrical properties of the matter between them. The capacitance is, therefore, independent of the voltage across the capacitor. $\boxed{(c)\text{ is correct.}}$

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Determine the Concept The capacitance of a parallel-plate capacitor is a function of the surface area of its plates, the separation of these plates, and the electrical properties of the matter between them. The capacitance is, therefore, independent of the charge of the capacitor. $\boxed{(c)\text{ is correct.}}$

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Picture the Problem The energy stored in the electric field of a parallel-plate capacitor is related to the potential difference across the capacitor by $U = \frac{1}{2}QV$.

Relate the potential energy stored in the electric field of the capacitor to the potential difference across the capacitor:

$$U = \tfrac{1}{2}QV$$

$$\boxed{\text{With } Q \text{ constant, } U \text{ is directly proportional to } V\text{. Hence, doubling } V \text{ doubles } U.}$$

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Picture the Problem The energy stored in a capacitor is given by $U = \frac{1}{2}QV$ and the capacitance of a parallel-plate capacitor by $C = \epsilon_0 A/d$. We can combine these relationships, using the definition of capacitance and the condition that the potential difference across the capacitor is constant, to express U as a function of d.

Express the energy stored in the capacitor:

$$U = \tfrac{1}{2}QV$$

Use the definition of capacitance to express the charge of the capacitor:

$$Q = CV$$

Substitute to obtain:

$$U = \tfrac{1}{2}CV^2$$

Express the capacitance of a parallel-plate capacitor in terms of the separation d of its plates:

$$C = \frac{\epsilon_0 A}{d}$$

where A is the area of one plate.

Substitute to obtain:

$$U = \frac{\epsilon_0 A V^2}{2d}$$

Because $U \propto \frac{1}{d}$, doubling the separation of the plates will reduce the energy stored in the capacitor to 1/2 its previous value:

$$\boxed{(d)\text{ is correct.}}$$

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Picture the Problem Let V represent the initial potential difference between the plates, U the energy stored in the capacitor initially, d the initial separation of the plates, and V', U', and d' these physical quantities when the plate separation has been doubled. We can use $U = \frac{1}{2}QV$ to relate the energy stored in the capacitor to the potential difference across it and $V = Ed$ to relate the potential difference to the separation of the plates.

Express the energy stored in the capacitor before the doubling of the separation of the plates:

$$U = \tfrac{1}{2}QV$$

Express the energy stored in the capacitor after the doubling of the separation of the plates:

$$U' = \tfrac{1}{2}QV'$$

because the charge on the plates does not change.

Express the ratio of U' to U:

$$\frac{U'}{U} = \frac{V'}{V}$$

Express the potential differences across the capacitor plates before and after the plate separation in terms of the electric field E between the plates:

$$V = Ed$$

and

$$V' = Ed'$$

because E depends solely on the charge on the plates and, as observed above, the charge does not change during the separation process.

Substitute to obtain: $\frac{U'}{U} = \frac{Ed'}{Ed} = \frac{d'}{d}$

For $d' = 2d$: $\frac{U'}{U} = \frac{2d}{d} = 2$ and $\boxed{(b) \text{ is correct}}$

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(*a*) False. The capacitance of a parallel-plate capacitor is defined to be the ratio of the charge on the capacitor to the potential difference across it.

(*b*) False. The capacitance of a parallel-plate capacitor depends on the area of its plates *A*, their separation *d*, and the dielectric constant κ of the material between the plates according to $C = \kappa \in_0 A/d$.

(*c*) False. As in part (*b*), the capacitance of a parallel-plate capacitor depends on the area of its plates *A*, their separation *d*, and the dielectric constant κ of the material between the plates according to $C = \kappa \in_0 A/d$.

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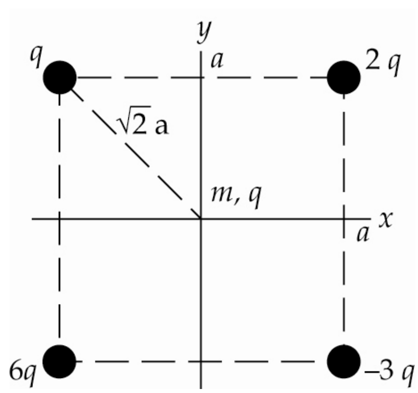
Picture the Problem The diagram shows the four charges fixed at the corners of the square and the fifth charge that is released from rest at the origin. We can use conservation of energy to relate the initial potential energy of the fifth particle to its kinetic energy when it is at a great distance from the origin and the electrostatic potential at the origin to express U_i.



Use conservation of energy to relate the initial potential energy of the particle to its kinetic energy when it is at a great distance from the origin:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

Express the initial potential energy of the particle to its charge and the electrostatic potential at the origin:

$$U_i = qV(0)$$

Substitute for K_f and U_i to obtain:

$$\tfrac{1}{2}mv^2 - qV(0) = 0$$

Solve for *v*:

$$v = \sqrt{\frac{2qV(0)}{m}}$$

Express the electrostatic potential at the origin:

$$V(0) = \frac{kq}{\sqrt{2}a} + \frac{2kq}{\sqrt{2}a} + \frac{-3kq}{\sqrt{2}a} + \frac{6kq}{\sqrt{2}a}$$
$$= \frac{6kq}{\sqrt{2}a}$$

Substitute and simplify to obtain:

$$v = \sqrt{\frac{2q}{m}\left(\frac{6kq}{\sqrt{2}a}\right)} = \boxed{q\sqrt{\frac{6\sqrt{2}k}{ma}}}$$

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Picture the Problem We can use the definition of capacitance and the expression for the potential difference between charged concentric spherical shells to show that $C = 4\pi \in_0 R_1R_2/(R_2 - R_1)$.

(*a*) Using its definition, relate the capacitance of the concentric spherical shells to their charge *Q* and the potential difference *V* between their surfaces:

$$C = \frac{Q}{V}$$

Express the potential difference between the conductors:

$$V = kQ\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = kQ\frac{R_2 - R_1}{R_1R_2}$$

Substitute to obtain:

$$C = \frac{Q}{kQ\frac{R_2 - R_1}{R_1R_2}} = \frac{R_1R_2}{k(R_2 - R_1)}$$
$$= \boxed{\frac{4\pi \in_0 R_1R_2}{R_2 - R_1}}$$

(*b*) Because $R_2 = R_1 + d$:

$$R_1R_2 = R_1(R_1 + d)$$
$$= R_1^2 + R_1d$$
$$\approx R_1^2 = R^2$$

because *d* is small.

Substitute to obtain:

$$C \approx \frac{4\pi \in_0 R^2}{d} = \boxed{\frac{\in_0 A}{d}}$$