# HOMEWORK#4 SOLUTION

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**Picture the Problem** A charged particle placed in an electric field experiences an accelerating force that does work on the particle. From the work-kinetic energy theorem we know that the work done on the particle by the net force changes its kinetic energy and that the kinetic energy $K$ acquired by such a particle whose charge is $q$ that is accelerated through a potential difference $V$ is given by $K = qV$. Let the numeral 1 refer to the alpha particle and the numeral 2 to the lithium nucleus and equate their kinetic energies after being accelerated through potential differences $V_1$ and $V_2$.

Express the kinetic energy of the alpha particle when it has been accelerated through a potential difference $V_1$:

$$K_1 = q_1 V_1 = 2eV_1$$

Express the kinetic energy of the lithium nucleus when it has been accelerated through a potential difference $V_2$:

$$K_2 = q_2 V_2 = 3eV_2$$

Equate the kinetic energies to obtain:

$$2eV_1 = 3eV_2$$

or

$$V_2 = \tfrac{2}{3}V_1 \text{ and } \boxed{(b) \text{ is correct.}}$$

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**Picture the Problem** We can use Coulomb's law and the superposition of fields to find $E$ at the origin and the definition of the electric potential due to a point charge to find $V$ at the origin.

Apply Coulomb's law and the superposition of fields to find the electric field $E$ at the origin:

$$\vec{E} = \vec{E}_{+Q \text{ at } -a} + \vec{E}_{+Q \text{ at } a} = \frac{kQ}{a^2}\hat{\boldsymbol{i}} - \frac{kQ}{a^2}\hat{\boldsymbol{i}} = 0$$

Express the potential $V$ at the origin:

$$V = V_{+Q \text{ at } -a} + V_{+Q \text{ at } a} = \frac{kQ}{a} + \frac{kQ}{a} = \frac{2kQ}{a}$$

and $\boxed{(b) \text{ is correct.}}$

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**Picture the Problem** We can use the definition of finite potential difference to find the potential difference $V(4\text{ m}) - V(0)$ and conservation of energy to find the kinetic energy of the charge when it is at $x = 4$ m. We can also find $V(x)$ if $V(x)$ is assigned various values at various positions from the definition of finite potential difference.

(*a*) Apply the definition of finite potential difference to obtain:

$$V(4\text{ m}) - V(0) = -\int_a^b \vec{E}\cdot d\vec{\ell} = -\int_0^{4\text{ m}} E\,d\ell = -(2\text{ kN/C})(4\text{ m}) = \boxed{-8.00\text{ kV}}$$

(*b*) By definition, $\Delta U$ is given by:

$$\Delta U = q\Delta V = (3\,\mu\text{C})(-8\text{ kV}) = \boxed{-24.0\text{ mJ}}$$

(*c*) Use conservation of energy to relate $\Delta U$ and $\Delta K$:

$$\Delta K + \Delta U = 0$$

or

$$K_{4\text{ m}} - K_0 + \Delta U = 0$$

Because $K_0 = 0$:

$$K_{4\text{ m}} = -\Delta U = \boxed{24.0\text{ mJ}}$$

Use the definition of finite potential difference to obtain:

$$V(x) - V(x_0) = -E_x(x - x_0) = -(2\text{ kV/m})(x - x_0)$$

(*d*) For $V(0) = 0$:

$$V(x) - 0 = -(2\text{ kV/m})(x - 0)$$

or

$$V(x) = \boxed{-(2\text{ kV/m})x}$$

(*e*) For $V(0) = 4$ kV:

$$V(x) - 4\text{ kV} = -(2\text{ kV/m})(x - 0)$$

or

$$V(x) = \boxed{4\text{ kV} - (2\text{ kV/m})x}$$

(*f*) For $V(1\text{m}) = 0$:

$$V(x) - 0 = -(2\text{ kV/m})(x - 1)$$

or

$$V(x) = \boxed{2\text{ kV} - (2\text{ kV/m})x}$$

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**Picture the Problem** We know that energy is conserved in the interaction between the $\alpha$

particle and the massive nucleus. Under the assumption that the recoil of the massive nucleus is negligible, we know that the initial kinetic energy of the $\alpha$ particle will be transformed into potential energy of the two-body system when the particles are at their distance of closest approach.

(*a*) Apply conservation of energy to the system consisting of the $\alpha$ particle and the massive nucleus:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because $K_f = U_i = 0$ and $K_i = E$:

$$-E + U_f = 0$$

Letting $r$ be the separation of the particles at closest approach, express $U_f$:

$$U_f = \frac{kq_{\text{nucleus}}q_\alpha}{r} = \frac{k(Ze)(2e)}{r} = \frac{2kZe^2}{r}$$

Substitute to obtain:

$$-E + \frac{2kZe^2}{r} = 0$$

Solve for $r$ to obtain:

$$r = \boxed{\frac{2kZe^2}{E}}$$

(*b*) For a 5.0-MeV $\alpha$ particle and a gold nucleus:

$$r_5 = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(79)(1.6\times10^{-19}\ \text{C})^2}{(5\,\text{MeV})(1.6\times10^{-19}\ \text{J/eV})} = 4.55\times10^{-14}\ \text{m} = \boxed{45.4\,\text{fm}}$$

For a 9.0-MeV $\alpha$ particle and a gold nucleus:

$$r_9 = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(79)(1.6\times10^{-19}\ \text{C})^2}{(9\,\text{MeV})(1.6\times10^{-19}\ \text{J/eV})} = 2.53\times10^{-14}\ \text{m} = \boxed{25.3\,\text{fm}}$$

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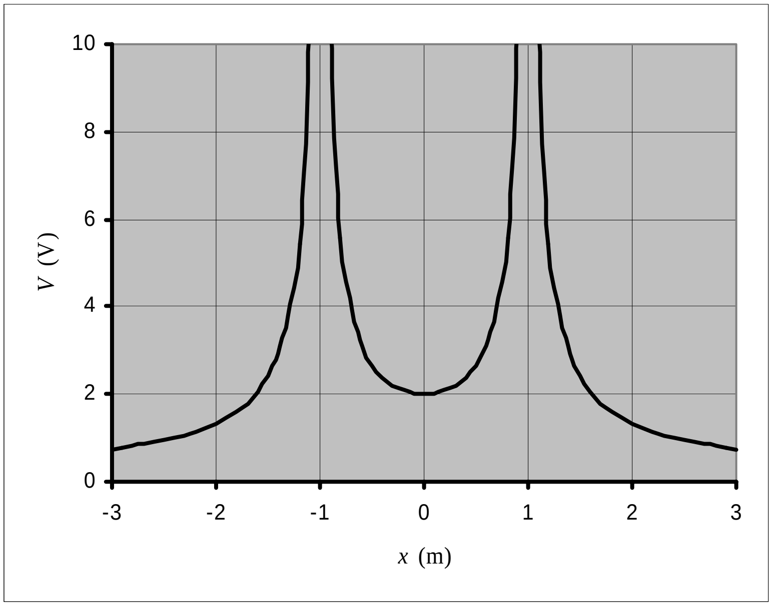
**Picture the Problem** For the two charges, $r = |x-a|$ and $|x+a|$ respectively and the electric potential at $x$ is the algebraic sum of the potentials at that point due to the charges at $x = +a$ and $x = -a$.

(*a*) Express $V(x)$ as the sum of the potentials due to the charges at $x = +a$ and $x = -a$:

$$V = \boxed{kq\left(\frac{1}{|x-a|} + \frac{1}{|x+a|}\right)}$$

(*b*) The following graph of $V(x)$ versus $x$ for $kq = 1$ and $a = 1$ was plotted using a spreadsheet program:




(*c*) At $x = 0$:

$$\frac{dV}{dx} = \boxed{0} \text{ and } E_x = -\frac{dV}{dx} = \boxed{0}$$

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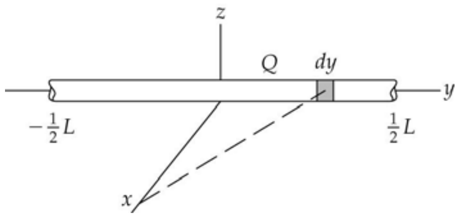
**Picture the Problem** Let the charge per unit length be $\lambda = Q/L$ and $dy$ be a line element with charge $\lambda dy$. We can express the potential $dV$ at any point on the $x$ axis due to $\lambda dy$ and integrate of find $V(x, 0)$.




(*a*) Express the element of potential $dV$ due to the line element $dy$:

$$dV = \frac{k\lambda}{r}dy$$

where $r = \sqrt{x^2 + y^2}$

Integrate $dV$ from $y = -L/2$ to $y = L/2$:

$$V(x,0) = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dy}{\sqrt{x^2 + y^2}}$$

$$= \boxed{\frac{kQ}{L}\ln\left(\frac{\sqrt{x^2 + L^2/4} + L/2}{\sqrt{x^2 + L^2/4} - L/2}\right)}$$

(*b*) Factor *x* from the numerator and denominator within the parentheses to obtain:

$$V(x,0)=\frac{kQ}{L}\ln\left(\frac{\sqrt{1+\frac{L^2}{4x^2}}+\frac{L}{2x}}{\sqrt{1+\frac{L^2}{4x^2}}-\frac{L}{2x}}\right)$$

Use $\ln\frac{a}{b}=\ln a-\ln b$ to obtain:

$$V(x,0)=\frac{kQ}{L}\left\{\ln\left(\sqrt{1+\frac{L^2}{4x^2}}+\frac{L}{2x}\right)-\ln\left(\sqrt{1+\frac{L^2}{4x^2}}-\frac{L}{2x}\right)\right\}$$

Let $\varepsilon=\frac{L^2}{4x^2}$ and use $(1+\varepsilon)^{1/2}=1+\frac{1}{2}\varepsilon-\frac{1}{8}\varepsilon^2+\ldots$ to expand $\sqrt{1+\frac{L^2}{4x^2}}$:

$$\left(1+\frac{L^2}{4x^2}\right)^{1/2}=1+\frac{1}{2}\frac{L^2}{4x^2}-\frac{1}{8}\left(\frac{L^2}{4x^2}\right)^2+\ldots\approx 1 \text{ for } x>>L.$$

Substitute to obtain:

$$V(x,0)=\frac{kQ}{L}\left\{\ln\left(1+\frac{L}{2x}\right)-\ln\left(1-\frac{L}{2x}\right)\right\}$$

Let $\delta=\frac{L}{2x}$ and use $\ln(1+\delta)=\delta-\frac{1}{2}\delta^2+\ldots$ to expand $\ln\left(1\pm\frac{L}{2x}\right)$:

$$\ln\left(1+\frac{L}{2x}\right)\approx\frac{L}{2x}-\frac{L^2}{4x^2} \text{ and } \ln\left(1-\frac{L}{2x}\right)\approx-\frac{L}{2x}-\frac{L^2}{4x^2} \text{ for } x>>L.$$

Substitute and simplify to obtain:

$$V(x,0)=\frac{kQ}{L}\left\{\frac{L}{2x}-\frac{L^2}{4x^2}-\left(-\frac{L}{2x}-\frac{L^2}{4x^2}\right)\right\}=\boxed{\frac{kQ}{x}}$$

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**Picture the Problem** We can use $W_{q\to\text{final position}}=q\Delta V_{i\to f}$ to find the work required to move these charges between the given points.

(*a*) Express the required work in terms of the charge being moved and the potential due to the charge at $x = +a$:

$$\begin{aligned} W_{+Q\to+a} &= Q\Delta V_{\infty\to+a} \\ &= Q[V(a)-V(\infty)] \\ &= QV(a) = Q\left(\frac{kQ}{2a}\right) = \boxed{\frac{kQ^2}{2a}} \end{aligned}$$

(*b*) Express the required work in terms of the charge being moved and the potentials due to the charges at $x = +a$ and $x = -a$:

$$\begin{aligned} W_{-Q\to0} &= -Q\Delta V_{\infty\to0} \\ &= -Q[V(0)-V(\infty)] \\ &= -QV(0) \\ &= -Q\left[V_{\text{charge at -}a} + V_{\text{charge at +}a}\right] \\ &= -Q\left(\frac{kQ}{a}+\frac{kQ}{a}\right) = \boxed{\frac{-2kQ^2}{a}} \end{aligned}$$

(*c*) Express the required work in terms of the charge being moved and the potentials due to the charges at $x = +a$ and $x = -a$:

$$\begin{aligned} W_{-Q\to2a} &= -Q\Delta V_{0\to2a} \\ &= -Q[V(2a)-V(0)] \\ &= -Q\left[V_{\text{charge at -}a} + V_{\text{charge at +}a} - V(0)\right] \\ &= -Q\left(\frac{kQ}{3a}+\frac{kQ}{a}-\frac{2kQ}{a}\right) \\ &= \boxed{\frac{2kQ^2}{3a}} \end{aligned}$$