# HOMEWORK#3 SOLUTION

**6** •

**Determine the Concept** Yes. The electric field on a closed surface is related to the net flux through it by Gauss's law: $\phi = \oint_S E dA = Q_{\text{inside}} / \epsilon_0$ . If the net flux through the closed surface is zero, the net charge inside the surface must be zero by Gauss's law.

**7** •

**Determine the Concept** The negative point charge at the center of the conducting shell induces a charge +$Q$ on the inner surface of the shell. $\boxed{(a)\text{ is correct.}}$

**8** •

**Determine the Concept** The negative point charge at the center of the conducting shell induces a charge +$Q$ on the inner surface of the shell. Because a conductor does not have to be neutral, $\boxed{(d)\text{ is correct.}}$

**12** ••

**Determine the Concept** No. The electric field on a closed surface is related to the net flux through it by Gauss's law: $\phi = \oint_S E dA = Q_{\text{inside}} / \epsilon_0$ . $\phi$ can be zero without $E$ being zero everywhere. If the net flux through the closed surface is zero, the net charge inside the surface must be zero by Gauss's law.

***36** •

**Picture the Problem** We'll define the flux of the gravitational field in a manner that is analogous to the definition of the flux of the electric field and then substitute for the gravitational field and evaluate the integral over the closed spherical surface.

Define the gravitational flux as:

$$\phi_g = \oint_S \vec{g} \cdot \hat{n} dA$$

Substitute for $\vec{g}$ and evaluate the integral to obtain:

$$\phi_g = \oint_S \left( -\frac{Gm}{r^2} \hat{r} \right) \cdot \hat{n} dA = -\frac{Gm}{r^2} \oint_S dA$$

$$= \left( -\frac{Gm}{r^2} \right) \left( 4\pi r^2 \right) = \boxed{-4\pi Gm}$$

**39** •

**Picture the Problem** To find $E_n$ in these three regions we can choose Gaussian surfaces of appropriate radii and apply Gauss's law. On each of these surfaces, $E_r$ is constant and Gauss's law relates $E_r$ to the total charge inside the surface.

(*a*) Use Gauss's law to find the electric field in the region $r < R_1$:

$$\oint_S E_n dA = \frac{1}{\in_0} Q_{\text{inside}}$$

and

$$E_{r<R_1} = \frac{Q_{\text{inside}}}{\in_0 A} = \boxed{0}$$

because $Q_{\text{inside}} = 0$.

Apply Gauss's law in the region $R_1 < r < R_2$:

$$E_{R_1<r<R_1} = \frac{q_1}{\in_0 (4\pi r^2)} = \boxed{\frac{kq_1}{r^2}}$$

Using Gauss's law, find the electric field in the region $r > R_2$:

$$E_{r>R_2} = \frac{q_1 + q_2}{\in_0 (4\pi r^2)} = \boxed{\frac{k(q_1 + q_2)}{r^2}}$$

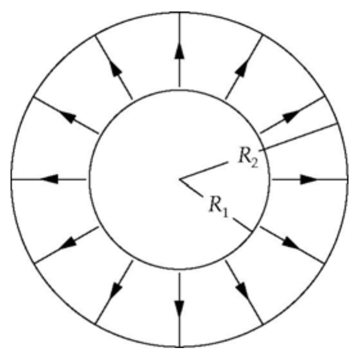
(*b*) Set $E_{r>R_2} = 0$ to obtain:

$$q_1 + q_2 = 0$$

or

$$\frac{q_1}{q_2} = \boxed{-1}$$

(*c*) The electric field lines for the situation in (*b*) with $q_1$ positive is shown to the right.




**44** ••

**Picture the Problem** We can find the total charge on the sphere by expressing the charge *dq* in a spherical shell and integrating this expression between $r = 0$ and $r = R$. By symmetry, the electric fields must be radial. To find $E_r$ inside the charged sphere we choose a spherical Gaussian surface of radius $r < R$. To find $E_r$ outside the charged sphere we choose a spherical Gaussian surface of radius $r > R$. On each of these surfaces, $E_r$ is constant. Gauss's law then relates $E_r$ to the total charge inside the surface.

(*a*) Express the charge *dq* in a shell of thickness *dr* and volume $4\pi r^2\, dr$:

$$dq = 4\pi r^2 \rho dr = 4\pi r^2 (Ar) dr$$
$$= 4\pi A r^3 dr$$

Integrate this expression from $r = 0$ to $R$ to find the total charge on the sphere:

$$Q = 4\pi A\int_0^R r^3 dr = \left[\pi A r^4\right]_0^R = \boxed{\pi A R^4}$$

($b$) Apply Gauss's law to a spherical surface of radius $r > R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\in_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\in_0}$$

Solve for $E_r$:

$$E_r(r > R) = \frac{Q_{\text{inside}}}{4\pi \in_0}\frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2}$$

$$= \frac{kA\pi R^4}{r^2} = \boxed{\frac{AR^4}{4\in_0 r^2}}$$

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\in_0} Q_{\text{inside}}$$

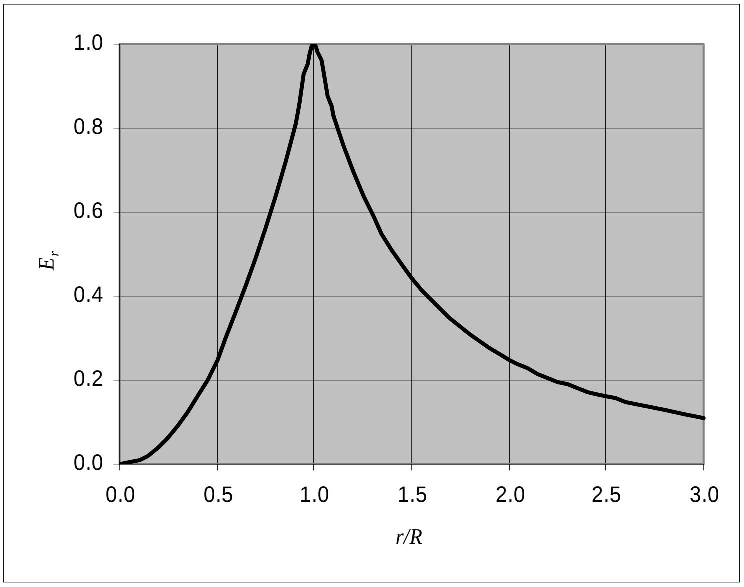
or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\in_0}$$

Solve for $E_r$:

$$E_r(r < R) = \frac{Q_{\text{inside}}}{4\pi r^2 \in_0} = \frac{\pi A r^4}{4\pi r^2 \in_0} = \boxed{\frac{Ar^2}{4\in_0}}$$

The graph of $E_r$ versus $r/R$, with $E_r$ in units of $A/4\in_0$, was plotted using a spreadsheet program.




**Remarks: Note that the results for (*a*) and (*b*) agree at *r* = *R*.**

**48** ••

**Picture the Problem** From symmetry, the field in the tangential direction must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long, uniformly charged cylindrical shell.

Apply Gauss's law to the cylindrical surface of radius *r* and length *L* that is concentric with the infinitely long, uniformly charged cylindrical shell:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for $E_n$:

$$E_n = \frac{Q_{\text{inside}}}{2\pi r L \epsilon_0} = \frac{2kQ_{\text{inside}}}{Lr}$$

For $r < R$, $Q_{\text{inside}} = 0$ and:

$$E_n(r < R) = \boxed{0}$$

For $r > R$, $Q_{\text{inside}} = \lambda L$ and:

$$E_n(r > R) = \frac{2k\lambda L}{Lr} = \frac{2k\lambda}{r} = \frac{2k(2\pi R\sigma)}{r}$$

$$= \boxed{\frac{R\sigma}{\epsilon_0 r}}$$

**61** •

**Picture the Problem** We can construct a Gaussian surface in the shape of a sphere of radius $r$ with the same center as the shell and apply Gauss's law to find the electric field as a function of the distance from this point. The inner and outer surfaces of the shell will have charges induced on them by the charge $q$ at the center of the shell.

(*a*) Apply Gauss's law to a spherical surface of radius $r$ that is concentric with the point charge:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{inside}$$

or

$$4\pi r^2 E_n = \frac{Q_{inside}}{\epsilon_0}$$

Solve for $E_n$:

$$E_n = \frac{Q_{inside}}{4\pi r^2 \epsilon_0} \qquad (1)$$

For $r < a$, $Q_{inside} = q$. Substitute in equation (1) and simplify to obtain:

$$E_n(r < a) = \frac{q}{4\pi r^2 \epsilon_0} = \boxed{\frac{kq}{r^2}}$$

Because the spherical shell is a conductor, a charge $-q$ will be induced on its inner surface. Hence, for $a < r < b$:

$$Q_{inside} = 0$$

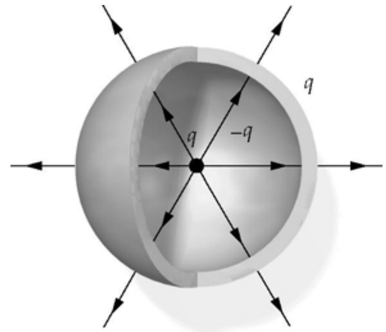
and

$$E_n(a < r < b) = \boxed{0}$$

For $r > b$, $Q_{inside} = q$. Substitute in equation (1) and simplify to obtain:

$$E_n(r > b) = \frac{q}{4\pi r^2 \epsilon_0} = \boxed{\frac{kq}{r^2}}$$

(*b*) The electric field lines are shown in the diagram to the right:




(*c*) A charge $-q$ is induced on the inner surface. Use the definition of surface charge density to obtain:

$$\sigma_{inner} = \frac{-q}{4\pi a^2} = \boxed{-\frac{q}{4\pi a^2}}$$

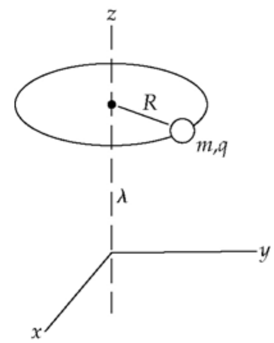
A charge $q$ is induced on the outer surface. Use the definition of surface

$$\sigma_{outer} = \boxed{\frac{q}{4\pi b^2}}$$

charge density to obtain:

**83** ••

**Picture the Problem** We can find the period of the motion from its angular frequency and apply Newton's 2nd law to relate $\omega$ to $m$, $q$, $R$, and the electric field due to the infinite line charge. Because the electric field is given by $E_r = 2k\lambda/r$ we can express $\omega$ and, hence, $T$ as a function of $m$, $q$, $R$, and $\lambda$.




Relate the period $T$ of the particle to its angular frequency $\omega$:

$$T = \frac{2\pi}{\omega} \qquad (1)$$

Apply Newton's 2nd law to the particle to obtain:

$$\sum F_{\text{radial}} = qE_r = mR\omega^2$$

Solve for $\omega$:

$$\omega = \sqrt{\frac{qE_r}{mR}}$$

Express the electric field at a distance $R$ from the infinite line charge:

$$E_r = 2k\frac{\lambda}{R}$$

Substitute in the expression for $\omega$:

$$\omega = \sqrt{\frac{2k\lambda q}{mR^2}} = \frac{1}{R}\sqrt{\frac{2k\lambda q}{m}}$$

Substitute in equation (1) to obtain:

$$T = \boxed{2\pi R\sqrt{\frac{m}{2k\lambda q}}}$$

**95** ••

**Picture the Problem** We can find the distance from the center where the net force on either charge is zero by setting the sum of the forces acting on either point charge equal to zero. Each point charge experiences two forces; one a Coulomb force of repulsion due to the other point charge, and the second due to that fraction of the sphere's charge that is between the point charge and the center of the sphere that creates an electric field at the location of the point charge.

Apply $\sum F = 0$ to either of the point charges:

$$F_{\text{Coulomb}} - F_{\text{field}} = 0 \qquad (1)$$

Express the Coulomb force on the proton:

$$F_{\text{Coulomb}} = \frac{ke^2}{(2a)^2} = \frac{ke^2}{4a^2}$$

The force exerted by the field $E$ is:

$$F_{\text{field}} = eE$$

Apply Gauss's law to a spherical surface of radius $a$ centered at the origin:

$$E\left(4\pi a^2\right) = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Relate the charge density of the electron sphere to $Q_{\text{enclosed}}$:

$$\frac{2e}{\frac{4}{3}\pi R^3} = \frac{Q_{\text{enclosed}}}{\frac{4}{3}\pi a^3} \Rightarrow Q_{\text{enclosed}} = \frac{2ea^3}{R^3}$$

Substitute for $Q_{\text{enclosed}}$:

$$E\left(4\pi a^2\right) = \frac{2ea^3}{\epsilon_0 R^3}$$

Solve for $E$ to obtain:

$$E = \frac{ea}{2\pi \epsilon_0 R^3} \Rightarrow F_{\text{field}} = \frac{e^2 a}{2\pi \epsilon_0 R^3}$$

Substitute for $F_{\text{Coulomb}}$ and $F_{\text{field}}$ in equation (1):

$$\frac{ke^2}{4a^2} - \frac{e^2 a}{2\pi \epsilon_0 R^3} = 0$$

or

$$\frac{ke^2}{4a^2} - \frac{2ke^2 a}{R^3} = 0$$

Solve for $a$ to obtain:

$$a = \sqrt[3]{\frac{1}{8}}R = \boxed{0.5R}$$

# HONORS SECTION

**56** •••

**Picture the Problem** From symmetry; the field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long nonconducting cylindrical shell.

Apply Gauss's law to a cylindrical surface of radius *r* and length *L* that is concentric with the infinitely long nonconducting cylindrical shell:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{inside}$$

or

$$2\pi r L E_n = \frac{Q_{inside}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for $E_n$:

$$E_n = \frac{Q_{inside}}{2\pi r L \epsilon_0}$$

For $r < a$, $Q_{inside} = 0$:

$$E_n(r < a) = \boxed{0}$$

Express $Q_{inside}$ for $a < r < b$:

$$Q_{inside} = \rho V = \rho\pi r^2 L - \rho\pi a^2 L = \rho\pi L(r^2 - a^2)$$

Substitute for $Q_{inside}$ to obtain:

$$E_n(a < r < b) = \frac{\rho\pi L(r^2 - a^2)}{2\pi \epsilon_0 L r} = \boxed{\frac{\rho(r^2 - a^2)}{2 \epsilon_0 r}}$$

Express $Q_{inside}$ for $r > b$:

$$Q_{inside} = \rho V = \rho\pi b^2 L - \rho\pi a^2 L = \rho\pi L(b^2 - a^2)$$

Substitute for $Q_{inside}$ to obtain:

$$E_n(r > b) = \frac{\rho\pi L(b^2 - a^2)}{2\pi \epsilon_0 r L} = \boxed{\frac{\rho(b^2 - a^2)}{2 \epsilon_0 r}}$$

**87** ••

**Picture the Problem** We can apply Gauss's law to express $\vec{E}$ as a function of *r*. We can use the hint to think of the fields at points 1 and 2 as the sum of the fields due to a sphere of radius *a* with a uniform charge distribution $\rho$ and a sphere of radius *b*, centered at *a*/2 with uniform charge distribution $-\rho$.

(*a*) The electric field at a distance *r* from the center of the sphere is given by:

$$\vec{E} = E\hat{r} \qquad (1)$$

where $\hat{r}$ is a unit vector pointing radially outward.

Apply Gauss's law to a spherical surface of radius *r* centered at the origin to obtain:

$$\oint_S E_n dA = E\left(4\pi r^2\right) = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Relate $Q_{\text{enclosed}}$ to the charge density $\rho$:

$$\rho = \frac{Q_{\text{enclosed}}}{\frac{4}{3}\pi r^3} \Rightarrow Q_{\text{enclosed}} = \tfrac{4}{3}\rho\pi r^3$$

Substitute for $Q_{\text{enclosed}}$:

$$E\left(4\pi r^2\right) = \frac{\frac{4}{3}\rho\pi r^3}{\epsilon_0}$$

Solve for *E* to obtain:

$$E = \frac{\rho r}{3\epsilon_0}$$

Substitute for *E* in equation (1) to obtain:

$$\vec{E} = \boxed{\frac{\rho}{3\epsilon_0} r\,\hat{r}}$$

(*b*) The electric field at point 1 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_1 = \vec{E}_\rho + \vec{E}_{-\rho} = E_\rho\hat{r} + E_{-\rho}\hat{r} \qquad (2)$$

Apply Gauss's law to relate the magnitude of the field due to the positive charge distribution to the charge enclosed by the sphere:

$$E_\rho\left(4\pi a^2\right) = \frac{q_{\text{encl}}}{\epsilon_0} = \frac{\frac{4}{3}\pi a^3\rho}{\epsilon_0}$$

Solve for $E_\rho$:

$$E_\rho = \frac{a\rho}{3\epsilon_0} = \frac{2\rho b}{3\epsilon_0}$$

Proceed similarly for the spherical hole to obtain:

$$E_{-\rho}\left(4\pi b^2\right) = \frac{q_{\text{encl}}}{\epsilon_0} = -\frac{\frac{4}{3}\pi b^3\rho}{\epsilon_0}$$

Solve for $E_{-\rho}$:

$$E_{-\rho} = -\frac{\rho b}{3\epsilon_0}$$

Substitute in equation (2) to obtain:

$$\vec{E}_1 = \frac{2\rho b}{3\in_0}\hat{\boldsymbol{r}} - \frac{\rho b}{3\in_0}\hat{\boldsymbol{r}} = \boxed{\frac{\rho b}{3\in_0}\hat{\boldsymbol{r}}}$$

The electric field at point 2 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_2 = \vec{E}_\rho + \vec{E}_{-\rho} = E_\rho\hat{\boldsymbol{r}} + E_{-\rho}\hat{\boldsymbol{r}} \qquad (3)$$

Because point 2 is at the center of the larger sphere:

$$E_\rho = 0$$

The magnitude of the field at point 2 due to the negative charge distribution is:

$$E_{-\rho} = \frac{\rho b}{3\in_0}$$

Substitute in equation (3) to obtain:

$$\vec{E}_2 = 0 + \frac{\rho b}{3\in_0}\hat{\boldsymbol{r}} = \boxed{\frac{\rho b}{3\in_0}\hat{\boldsymbol{r}}}$$