HOMEWORK#2 SOLUTION

Chapter 21

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Picture the Problem We can use constant-acceleration equations to express the *x* and *y* coordinates of the electron in terms of the parameter *t* and Newton's 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for *y* as a function of *x*, *q*, and *m*. We can decide whether the electron will strike the upper plate by finding the maximum value of its *y* coordinate. Should we find that it does not strike the upper plate, we can determine where it strikes the lower plate by setting $y(x) = 0$.

Express the *x* and *y* coordinates of the electron as functions of time:

$$x = (v_0 \cos\theta)t$$

and

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}a_y t^2$$

Apply Newton's 2nd law to relate the acceleration of the electron to the net force acting on it:

$$a_y = \frac{F_{\text{net},y}}{m_e} = \frac{eE_y}{m_e}$$

Substitute in the *y*-coordinate equation to obtain:

$$y = (v_0 \sin\theta)t - \frac{eE_y}{2m_e}t^2$$

Eliminate the parameter *t* between the two equations to obtain:

$$y(x) = (\tan\theta)x - \frac{eE_y}{2m_e v_0^2 \cos^2\theta}x^2 \quad (1)$$

To find y_{max}, set $dy/dx = 0$ for extrema:

$$\frac{dy}{dx} = \tan\theta - \frac{eE_y}{m_e v_0^2 \cos^2\theta}x' = 0\ \text{for extrema}$$

Solve for x' to obtain:

$$x' = \frac{m_e v_0^2 \sin 2\theta}{2eE_y} \quad \text{(See remark below.)}$$

Substitute x' in $y(x)$ and simplify to obtain y_{max}:

$$y_{\text{max}} = \frac{m_e v_0^2 \sin^2\theta}{2eE_y}$$

Substitute numerical values and evaluate y_{max}:

$$y_{max} = \frac{(9.11\times10^{-31}\text{ kg})(5\times10^{6}\text{ m/s})^2\sin^2 45°}{2(1.6\times10^{-19}\text{ C})(3.5\times10^{3}\text{ N/C})} = 1.02\text{ cm}$$

and, because the plates are separated by 2 cm, the electron does not strike the upper plate.

To determine where the electron will strike the lower plate, set $y = 0$ in equation (1) and solve for x to obtain:

$$x = \frac{m_e v_0^2 \sin 2\theta}{eE_y}$$

Substitute numerical values and evaluate x:

$$x = \frac{(9.11\times10^{-31}\text{ kg})(5\times10^{6}\text{ m/s})^2\sin 90°}{(1.6\times10^{-19}\text{ C})(3.5\times10^{3}\text{ N/C})} = \boxed{4.07\text{ cm}}$$

Remarks: *x′* is an extremum, i.e., either a maximum or a minimum. To show that it is a maximum we need to show that d^2y/dx^2, evaluated at *x′*, is negative. A simple alternative is to use your graphing calculator to show that the graph of *y*(*x*) is a maximum at *x′*. Yet another alternative is to recognize that, because equation (1) is quadratic and the coefficient of x^2 is negative, its graph is a parabola that opens downward.

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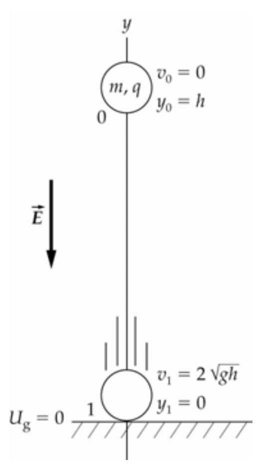
Picture the Problem Choose the coordinate system shown in the diagram and let $U_g = 0$ where $y = 0$. We'll let our system include the ball and the earth. Then the work done on the ball by the electric field will change the energy of the system. The diagram summarizes what we know about the motion of the ball. We can use the work-energy theorem to our system to relate the work done by the electric field to the change in its energy.



Using the work-energy theorem, relate the work done by the electric field to the change in the energy of the system:

$$W_{\text{electric field}} = \Delta K + \Delta U_g = K_2 - K_1 + U_{g,2} - U_{g,1}$$

or, because $K_1 = U_{g,2} = 0$,

$$W_{\text{electric field}} = K_2 - U_{g,1}$$

Substitute for $W_{\text{electric field}}$, K_2, and $U_{g,0}$ and simplify:

$$qEh = \tfrac{1}{2}mv_1^2 - mgh$$
$$= \tfrac{1}{2}m\left(2\sqrt{gh}\right)^2 - mgh = mgh$$

Solve for m:

$$m = \boxed{\frac{qE}{g}}$$

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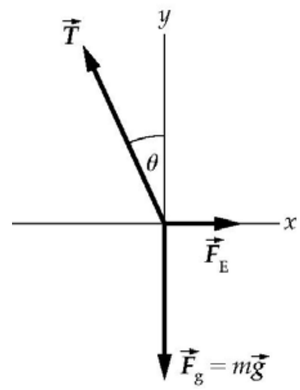
Picture the Problem Each sphere is in static equilibrium under the influence of the tension $\vec{T}$, the gravitational force $\vec{F}_g$, and the electric force $\vec{F}_E$. We can use Coulomb's law to relate the electric force to the charge on each sphere and their separation and the conditions for static equilibrium to relate these forces to the charge on each sphere.



(*a*) Apply the conditions for static equilibrium to the charged sphere:

$$\sum F_x = F_E - T\sin\theta = \frac{kq^2}{r^2} - T\sin\theta = 0$$

and

$$\sum F_y = T\cos\theta - mg = 0$$

Eliminate T between these equations to obtain:

$$\tan\theta = \frac{kq^2}{mgr^2}$$

Solve for q:

$$q = r\sqrt{\frac{mg\tan\theta}{k}}$$

Referring to the figure, relate the separation of the spheres r to the length of the pendulum L:

$$r = 2L\sin\theta$$

Substitute to obtain:

$$q = \boxed{2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}}}$$

(*b*) Evaluate q for m = 10 g, L = 50 cm, and θ = 10°:

$$q = 2(0.5\,\mathrm{m})\sin 10^\circ \sqrt{\frac{(0.01\,\mathrm{kg})(9.81\,\mathrm{m/s^2})\tan 10^\circ}{8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}}} = \boxed{0.241\,\mu\mathrm{C}}$$

Chapter 22

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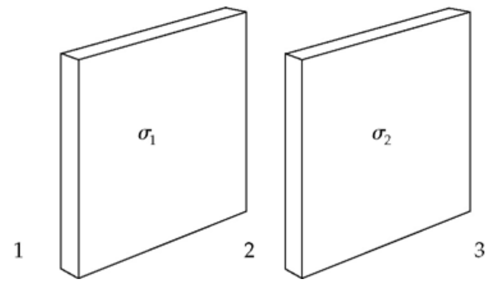
Picture the Problem Let the charge densities on the two plates be σ_1 and σ_2 and denote the three regions of interest as 1, 2, and 3. Choose a coordinate system in which the positive x direction is to the right. We can apply the equation for $\vec{E}$ near an infinite plane of charge and the superposition of fields to find the field in each of the three regions.



(a) Use the equation for $\vec{E}$ near an infinite plane of charge to express the field in region 1 when $\sigma_1 = \sigma_2 = +3\ \mu\text{C/m}^2$:

$$\begin{aligned}\vec{E}_1 &= \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2}\\ &= -2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i}\\ &= -4\pi k\sigma\,\hat{i}\end{aligned}$$

Substitute numerical values and evaluate $\vec{E}_1$:

$$\vec{E}_1 = -4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C/m}^2\right)\hat{i} = \boxed{-\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}$$

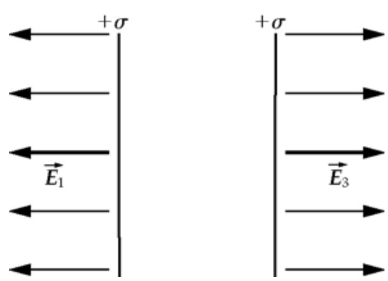
Proceed as above for region 2:

$$\begin{aligned}\vec{E}_2 &= \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i}\\ &= 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}\end{aligned}$$

Proceed as above for region 3:

$$\begin{aligned}\vec{E}_3 &= \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} + 2\pi k\sigma_2\hat{i}\\ &= 4\pi k\sigma\hat{i}\\ &= 4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C/m}^2\right)\hat{i}\\ &= \boxed{\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}\end{aligned}$$

The electric field lines are shown to the right:



(*b*) Use the equation for $\vec{E}$ near an infinite plane of charge to express and evaluate the field in region 1 when $\sigma_1 = +3\ \mu\text{C/m}^2$ and $\sigma_2 = -3\ \mu\text{C/m}^2$:

$$\vec{E}_1 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i}$$
$$= 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}$$

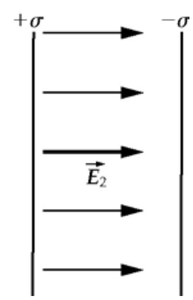
Proceed as above for region 2:

$$\vec{E}_1 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} + 2\pi k\sigma_2\hat{i}$$
$$= 4\pi k\sigma\hat{i}$$
$$= 4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C}\right)\hat{i}$$
$$= \boxed{\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}$$

Proceed as above for region 3:

$$\vec{E}_3 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i}$$
$$= 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}$$

The electric field lines are shown to the right:



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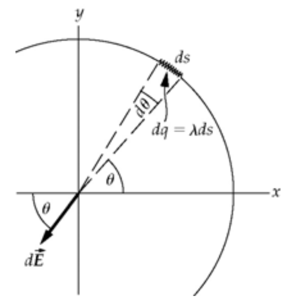
Picture the Problem The diagram shows a segment of the ring of length *ds* that has a charge $dq = \lambda ds$. We can express the electric field $d\vec{E}$ at the center of the ring due to the charge *dq* and then integrate this expression from $\theta = 0$ to 2π to find the magnitude of the field in the center of the ring.



(*a*) and (*b*) The field $d\vec{E}$ at the center of the ring due to the charge *dq* is:

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y = -dE\cos\theta\,\hat{i} - dE\sin\theta\,\hat{j} \qquad (1)$$

The magnitude *dE* of the field at the center of the ring is:

$$dE = \frac{kdq}{r^2}$$

Because $dq = \lambda ds$:

$$dE = \frac{k\lambda ds}{r^2}$$

The linear charge density varies with θ according to $\lambda(\theta) = \lambda_0 \sin\theta$:

$$dE = \frac{k\lambda_0 \sin\theta\, ds}{r^2}$$

Substitute $rd\theta$ for ds:

$$dE = \frac{k\lambda_0 \sin\theta\, rd\theta}{r^2} = \frac{k\lambda_0 \sin\theta\, d\theta}{r}$$

Substitute for dE in equation (1) to obtain:

$$d\vec{E} = -\frac{k\lambda_0 \sin\theta \cos\theta\, d\theta}{r}\hat{\boldsymbol{i}} - \frac{k\lambda_0 \sin^2\theta\, d\theta}{r}\hat{\boldsymbol{j}}$$

Integrate $d\vec{E}$ from $\theta = 0$ to 2π:

$$\begin{aligned}\vec{E} &= -\frac{k\lambda_0}{2r}\int_0^{2\pi}\sin 2\theta\, d\theta\,\hat{\boldsymbol{i}} - \frac{k\lambda_0}{r}\int_0^{2\pi}\sin^2\theta\, d\theta\,\hat{\boldsymbol{j}}\\ &= 0 - \frac{\pi k\lambda_0}{r}\hat{\boldsymbol{j}}\\ &= \boxed{-\frac{\pi k\lambda_0}{r}\hat{\boldsymbol{j}}}\end{aligned}$$

$$\boxed{\text{The field at the origin is in the negative } y \text{ direction and its magnitude is } \frac{\pi k\lambda_0}{r}.}$$

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Picture the Problem In parts (*a*) and (*b*) we can express the charges on each of the elements as the product of the linear charge density of the ring and the length of the segments. Because the lengths of the segments are the product of the angle subtended at *P* and their distances from *P*, we can express the charges in terms of their distances from *P*. By expressing the ratio of the fields due to the charges on s_1 and s_2 we can determine their dependence on r_1 and r_2 and, hence, the resultant field at *P*. We can proceed similarly in part (*c*) with *E* varying as $1/r$ rather than $1/r^2$. In part (*d*), with s_1 and s_2 representing areas, we'll use the definition of the solid angle subtended by these areas to relate their charges to their distances from point *P*.

(*a*) Express the charge q_1 on the element of length s_1:

$$q_1 = \lambda s_1 = \lambda\theta r_1$$

where θ is the angle subtended by the arcs of length s_1 and s_2.

Express the charge q_2 on the element of length s_2:

$$q_2 = \lambda s_2 = \lambda\theta r_2$$

Divide the first of these equations by the second to obtain:

$$\frac{q_1}{q_2} = \frac{\lambda\theta r_1}{\lambda\theta r_2} = \boxed{\frac{r_1}{r_2}}$$

Express the electric field at P due to the charge associated with the element of length s_1:

$$E_1 = \frac{kq_1}{r_1^2} = \frac{k\lambda s_1}{r_1^2} = \frac{k\lambda\theta r_1}{r_1^2} = \frac{k\lambda\theta}{r_1}$$

Express the electric field at P due to the charge associated with the element of length s_2:

$$E_2 = \frac{k\lambda\theta}{r_2}$$

Divide the first of these equations by the second to obtain:

$$\frac{E_1}{E_2} = \frac{\dfrac{k\lambda\theta}{r_1}}{\dfrac{k\lambda\theta}{r_2}} = \frac{r_2}{r_1}$$

and, because $r_2 > r_1$,

$$\boxed{E_1 > E_2}$$

(*b*) The two fields point away from their segments of arc.

$$\boxed{\text{Because } E_1 > E_2, \text{ the resultant field points toward } s_2.}$$

(*c*) If E varies as $1/r$:

$$E_1 = \frac{kq_1}{r_1} = \frac{k\lambda s_1}{r_1} = \frac{k\lambda\theta r_1}{r_1} = k\lambda\theta$$

and

$$E_2 = \frac{kq_2}{r_2} = \frac{k\lambda s_2}{r_2} = \frac{k\lambda\theta r_2}{r_2} = k\lambda\theta$$

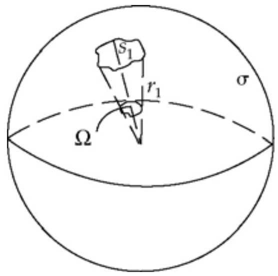
Therefore:

$$\boxed{E_1 = E_2}$$

(*d*) Use the definition of the solid angle Ω subtended by the area s_1 to obtain:

$$\frac{\Omega}{4\pi} = \frac{s_1}{4\pi r_1^2}$$

or



$$s_1 = \Omega r_1^2$$

Express the charge q_1 of the area s_1:

$$q_1 = \sigma s_1 = \sigma\Omega r_1^2$$

Similarly, for an element of area s_2:

$$s_2 = \Omega r_2^2$$
and
$$q_2 = \sigma\Omega r_2^2$$

Express the ratio of q_1 to q_2 to obtain:

$$\frac{q_1}{q_2} = \frac{\sigma\Omega r_1^2}{\sigma\Omega r_2^2} = \boxed{\frac{r_1^2}{r_2^2}}$$

Proceed as in (a) to obtain:

$$\frac{E_1}{E_2} = \frac{\dfrac{kq_1}{r_1^2}}{\dfrac{kq_2}{r_2^2}} = \frac{r_2^2 q_1}{r_1^2 q_2} = \frac{r_2^2\sigma\Omega r_1^2}{r_1^2\sigma\Omega r_2^2} = 1$$

Because the two fields are of equal magnitude and oppositely directed:

$$\boxed{\vec{E} = 0}$$

If $E \propto 1/r$, then s_2 would produce the stronger field at P and $\vec{E}$ would point toward s_1.

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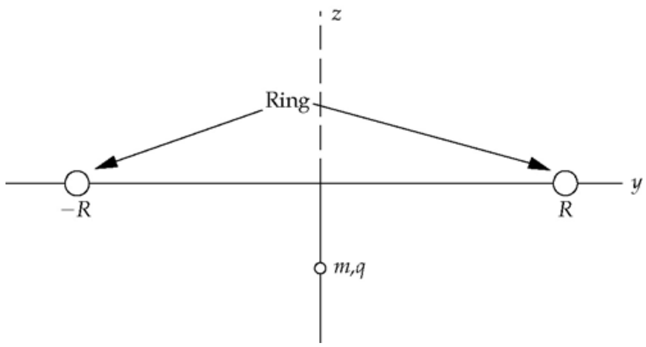
Picture the Problem We can apply the condition for translational equilibrium to the particle and use the expression for the electric field on the axis of a ring charge to obtain an expression for $|q|/m$. Doing so will lead us to the conclusion that $|q|/m$ will be a minimum when E_z is a maximum and so we'll use the result from Problem 26 that $z = -R/\sqrt{2}$ maximizes E_z.



(a) Apply $\sum F_z = 0$ to the particle:

$$|q|E_z - mg = 0$$

Solve for $|q|/m$:

$$\frac{|q|}{m} = \frac{g}{E_z} \qquad (1)$$

Note that this result tells us that the minimum value of $|q|/m$ will be where the field due to the ring is greatest.

Express the electric field along the z axis due to the ring of charge:

$$E_z = \frac{kQz}{\left(z^2 + R^2\right)^{3/2}}$$

Differentiate this expression with respect to z to obtain:

$$\frac{dE_x}{dz} = kQ\frac{d}{dz}\left[\frac{x}{\left(z^2+R^2\right)^{3/2}}\right] = kQ\frac{\left(z^2+R^2\right)^{3/2} - z\frac{d}{dx}\left(z^2+R^2\right)^{3/2}}{\left(z^2+R^2\right)^3}$$

$$= kQ\frac{\left(z^2+R^2\right)^{3/2} - z\left(\tfrac{3}{2}\right)\left(z^2+R^2\right)^{1/2}(2z)}{\left(z^2+R^2\right)^3} = kQ\frac{\left(z^2+R^2\right)^{3/2} - 3z^2\left(z^2+R^2\right)^{1/2}}{\left(z^2+R^2\right)^3}$$

Set this expression equal to zero for extrema and simplify:

$$\frac{\left(z^2+R^2\right)^{3/2} - 3z^2\left(z^2+R^2\right)^{1/2}}{\left(z^2+R^2\right)^3} = 0\,,$$

$$\left(z^2+R^2\right)^{3/2} - 3z^2\left(z^2+R^2\right)^{1/2} = 0\,,$$

and

$$z^2 + R^2 - 3z^2 = 0$$

Solve for x to obtain:

$$z = \pm\frac{R}{\sqrt{2}}$$

as candidates for maxima or minima.

You can either plot a graph of E_z or evaluate its second derivative at these points to show that it is a maximum at:

$$z = -\frac{R}{\sqrt{2}}$$

Substitute to obtain an expression $E_{z,\max}$:

$$E_{z,\max} = \left| \frac{kQ\left(-\frac{R}{\sqrt{2}}\right)}{\left(\left(-\frac{R}{\sqrt{2}}\right)^2 + R^2\right)^{3/2}} \right| = \frac{2kQ}{\sqrt{27}R^2}$$

Substitute in equation (1) to obtain:

$$\frac{|q|}{m} = \boxed{\frac{\sqrt{27}gR^2}{2kQ}}$$

(*b*) If $|q|/m$ is twice as great as in (*a*), then the electric field should be half its value in (*a*), i.e.:

$$\frac{kQ}{\sqrt{27}R^2} = \frac{kQz}{\left(z^2 + R^2\right)^{3/2}}$$

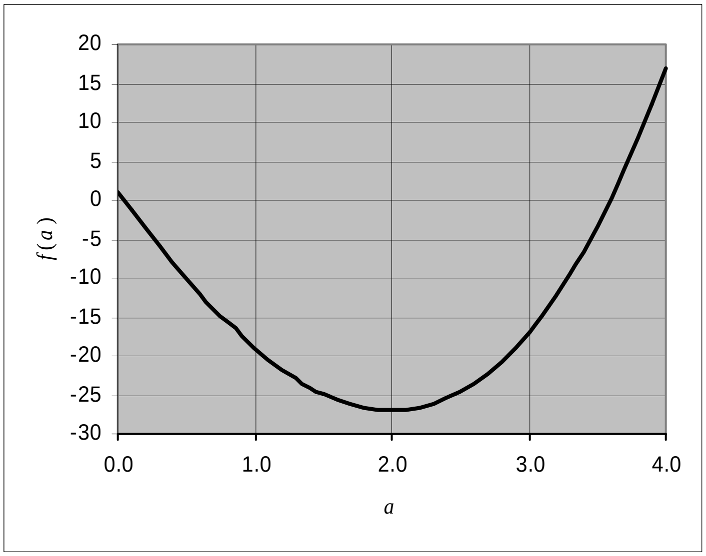
or

$$\frac{1}{27R^4} = \frac{z^2}{R^6\left(1 + \frac{z^2}{R^2}\right)^3}$$

Let $a = z^2/R^2$ and simplify to obtain:

$$a^3 + 3a^2 - 24a + 1 = 0$$

The graph of $f(a) = a^3 + 3a^2 - 24a + 1$ shown below was plotted using a spreadsheet program.

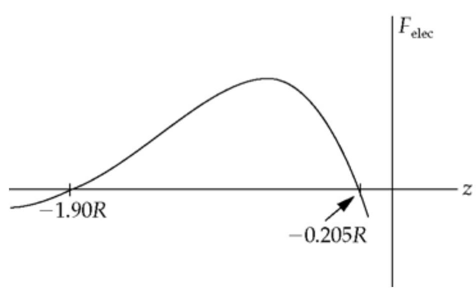


Use your calculator or trial-and-error methods to obtain: $a = 0.04188$ and $a = 3.596$

The corresponding z values are: $z = -0.205R$ and $z = -1.90R$

The condition for a stable equilibrium position is that the particle, when displaced from its equilibrium position, experiences a restoring force, i.e. a force that acts toward the equilibrium position. When the particle in this problem is just above its equilibrium position the net force on it must be downward and when it is just below the equilibrium position the net force on it must be upward. Note that the electric force is zero at the origin, so the net force there is downward and remains downward to the first equilibrium position as the weight force exceeds the electric force in this interval. The net force is upward between the first and second equilibrium positions as the electric force exceeds the weight force. The net force is downward below the second equilibrium position as the weight force exceeds the electric force. Thus, the first (higher) equilibrium position is stable and the second (lower) equilibrium position is unstable.

You might also find it instructive to use your graphing calculator to plot a graph of the electric force (the gravitational force is constant and only shifts the graph of the total force downward). Doing so will produce a graph similar to the one shown in the sketch to the right.



Note that the slope of the graph is negative on both sides of $-0.205R$ whereas it is positive on both sides of $-1.90R$; further evidence that $-0.205R$ is a position of stable equilibrium and $-1.90R$ a position of unstable equilibrium.

HONORS SECTION

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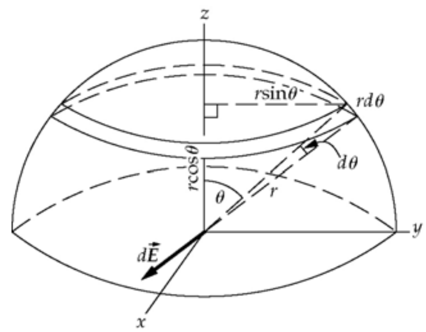
Picture the Problem Consider the ring with its axis along the *z* direction shown in the diagram. Its radius is $z = r\cos\theta$ and its width is $rd\theta$. We can use the equation for the field on the axis of a ring charge and then integrate to express the field at the center of the hemispherical shell.



Express the field on the axis of the ring charge:

$$dE = \frac{kzdq}{\left(r^2\sin^2\theta + r^2\cos^2\theta\right)^{3/2}} = \frac{kzdq}{r^3}$$

where $z = r\cos\theta$

Express the charge *dq* on the ring:

$$dq = \sigma dA = \sigma\left(2\pi r\sin\theta\right)rd\theta = 2\pi\sigma r^2\sin\theta d\theta$$

Substitute to obtain:

$$dE = \frac{k\left(r\cos\theta\right)2\pi\sigma r^2\sin\theta d\theta}{r^3} = 2\pi k\sigma\sin\theta\cos\theta d\theta$$

Integrate *dE* from $\theta = 0$ to $\pi/2$ to obtain:

$$E = 2\pi k\sigma\int_0^{\pi/2}\sin\theta\cos\theta d\theta = 2\pi k\sigma\left[\tfrac{1}{2}\sin^2\theta\right]_0^{\pi/2} = \boxed{\pi k\sigma}$$