HOMEWORK#12 SOLUTION

[Chap30]

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Picture the Problem We can differentiate the expression for the electric field between the plates of a parallel-plate capacitor to find the rate of change of the electric field and the definitions of the conduction current and electric flux to compute I_d.

(*a*) Express the electric field between the plates of the parallel-plate capacitor:

$$E = \frac{Q}{\epsilon_0 A}$$

Differentiate this expression with respect to time to obtain an expression for the rate of change of the electric field:

$$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{Q}{\epsilon_0 A}\right] = \frac{1}{\epsilon_0 A}\frac{dQ}{dt} = \frac{I}{\epsilon_0 A}$$

Substitute numerical values and evaluate dE/dt:

$$\frac{dE}{dt} = \frac{5\,\text{A}}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\pi\left(0.023\,\text{m}\right)^2} = \boxed{3.40\times10^{14}\,\text{V/m}\cdot\text{s}}$$

(*b*) Express the displacement current I_d:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt}$$

Substitute for the electric flux to obtain:

$$I_d = \epsilon_0 \frac{d}{dt}[EA] = \epsilon_0 A \frac{dE}{dt}$$

Substitute numerical values and evaluate I_d:

$$I_d = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\pi\left(0.023\,\text{m}\right)^2\left(3.40\times10^{14}\,\text{V/m}\cdot\text{s}\right) = \boxed{5.00\,\text{A}}$$

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Picture the Problem We can use Ampere's law to a circular path of radius *r* between the plates and parallel to their surfaces to obtain an expression relating *B* to the current enclosed by the amperian loop. Assuming that the displacement current is uniformly distributed between the plates, we can relate the displacement current enclosed by the circular loop to the conduction current *I*.

Apply Ampere's law to a circular path of radius *r* between the plates and parallel to their surfaces to obtain:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\pi r B = \mu_0 I_{\text{enclosed}} = \mu_0 I$$

Assuming that the displacement current is uniformly distributed:

$$\frac{I}{\pi r^2} = \frac{I_d}{\pi R^2} \Rightarrow I = \frac{r^2}{R^2} I_d$$

where *R* is the radius of the circular plates.

Substitute to obtain:

$$2\pi r B = \frac{\mu_0 r^2}{R^2} I_d$$

Solve for *B*:

$$B = \frac{\mu_0 r}{2\pi R^2} I_d$$

Substitute numerical values and evaluate *B*:

$$B(r) = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(5\ \text{A})}{2\pi\,(0.023\ \text{m})^2} r = \boxed{\left(1.89\times10^{-3}\ \text{T/m}\right)r}$$

TA's note: $I_{enclosed\,(displacement)} \neq I_d$ in this solution. I_d of this problem is the same value as I_d of the problem #15.