HOMEWORK#11 SOLUTION

[Chap29]

***1** •

Determine the Concept Because the rms current through the resistor is given by $I_{\text{rms}} = \mathcal{E}_{\text{rms}}/R$ and both ε_{rms} and R are independent of frequency, $\boxed{(b)\text{ is correct.}}$

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Picture the Problem We can use the relationship between V and V_{peak} to decide the effect of doubling the rms voltage on the peak voltage.

Express the initial rms voltage in terms of the peak voltage:

$$V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}}$$

Express the doubled rms voltage in terms of the new peak voltage V'_{max}:

$$2V_{\text{rms}} = \frac{V'_{\text{max}}}{\sqrt{2}}$$

Divide the second of these equations by the first and simplify to obtain:

$$\frac{2V_{\text{rms}}}{V_{\text{rms}}} = \frac{\dfrac{V'_{\text{max}}}{\sqrt{2}}}{\dfrac{V_{\text{max}}}{\sqrt{2}}} \text{ or } 2 = \frac{V'_{\text{max}}}{V_{\text{max}}}$$

Solve for V'_{max}:

$$V'_{\text{max}} = 2V_{\text{max}} \text{ and } \boxed{(a)\text{ is correct.}}$$

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Picture the Problem We can use $T = 2\pi/\omega$ and $\omega = 1/\sqrt{LC}$ to relate T (and hence f) to L and C.

(*a*) Express the period of oscillation of the *LC* circuit:

$$T = \frac{2\pi}{\omega}$$

For an *LC* circuit:

$$\omega = \frac{1}{\sqrt{LC}}$$

Substitute to obtain:

$$T = 2\pi\sqrt{LC} \qquad (1)$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{(2\,\text{mH})(20\,\mu\text{F})} = \boxed{1.26\,\text{ms}}$$

(*b*) Solve equation (1) for L to obtain:

$$L = \frac{T^2}{4\pi^2 C} = \frac{1}{4\pi^2 f^2 C}$$

Substitute numerical values and evaluate L:

$$L = \frac{1}{4\pi^2 \left(60\,\text{s}^{-1}\right)^2 \left(80\,\mu\text{F}\right)} = \boxed{88.0\,\text{mH}}$$

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Picture the Problem We can use the expression $f_0 = 1/2\pi\sqrt{LC}$ for the resonance frequency of an LC circuit to show that each circuit oscillates with the same frequency. In (*b*) we can use $I_{max} = \omega Q_0$, where Q_0 is the charge of the capacitor at time zero, and the definition of capacitance $Q_0 = CV$ to express I_{max} in terms of ω, C and V.

Express the resonance frequency for an LC circuit:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(*a*) Express the product of L and C for each circuit:

Circuit 1: L_1C_1,
Circuit 2: $L_2C_2 = \left(2L_1\right)\left(\tfrac{1}{2}C_1\right) = L_1C_1$,
and
Circuit 3: $L_3C_3 = \left(\tfrac{1}{2}L_1\right)\left(2C_1\right) = L_1C_1$

$$\boxed{\text{Because } L_1C_1 = L_2C_2 = L_3C_3\text{, the resonance frequencies of the three circuits are the same.}}$$

(*b*) Express I_{max} in terms of the charge stored in the capacitor:

$$I_{max} = \omega Q_0$$

Express Q_0 in terms of the capacitance of the capacitor and the potential difference across the capacitor:

$$Q_0 = CV$$

Substitute to obtain:

$$I_{max} = \omega CV$$

or, for ω and V constant,

$$I_{max} \propto C$$

$$\boxed{\text{The circuit with } C = C_3 \text{ has the greatest } I_{max}.}$$

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Picture the Problem We can apply Kirchhoff's loop rule to obtain expressions for I_R and I_L and then use trigonometric identities to show that $I = I_R + I_L = I_{max} \cos(\omega t - \delta)$, where $\tan\delta = R/X_L$ and $I_{max} = \varepsilon_{max}/Z$ with $Z^{-2} = R^{-2} + X_L^{-2}$.

(*a*) Apply Kirchhoff's loop rule to a clockwise loop that includes the source and the resistor:

$$\mathcal{E}_{max}\cos\omega t - I_R R = 0$$

Solve for I_R:

$$I_R = \boxed{\frac{\mathcal{E}_{max}}{R}\cos\omega t}$$

(*b*) Apply Kirchhoff's loop rule to a clockwise loop that includes the source and the inductor:

$$\mathcal{E}_{max}\cos(\omega t - 90°) - I_L X_L = 0$$

because the current lags the potential difference across the inductor by 90°.

Solve for I_L:

$$I_L = \boxed{\frac{\mathcal{E}_{max}}{X_L}\cos(\omega t - 90°)}$$

(*c*) Express the current drawn from the source in terms of I_{max} and the phase constant δ:

$$I = I_R + I_L = I_{max}\cos(\omega t - \delta)$$

Use a trigonometric identity to expand $\cos(\omega t - \delta)$:

$$I = I_{max}(\cos\omega t\cos\delta + \sin\omega t\sin\delta)$$
$$= I_{max}\cos\omega t\cos\delta + I_{max}\sin\omega t\sin\delta$$

From our results in (*a*):

$$I = I_R + I_L = \frac{\mathcal{E}_{max}}{R}\cos\omega t + \frac{\mathcal{E}_{max}}{X_L}\cos(\omega t - 90°)$$
$$= \frac{\mathcal{E}_{max}}{R}\cos\omega t + \frac{\mathcal{E}_{max}}{X_L}\sin\omega t$$

A useful trigonometric identity is:

$$A\cos\omega t + B\sin\omega t = \sqrt{A^2 + B^2}\cos(\omega t - \delta)$$

where

$$\delta = \tan^{-1}\frac{B}{A}$$

Apply this identity to obtain:

$$I=\sqrt{\left(\frac{\mathcal{E}_{\max}}{R}\right)^2+\left(\frac{\mathcal{E}_{\max}}{X_L}\right)^2}\cos(\omega t-\delta)\ (1)$$

and

$$\delta=\tan^{-1}\left(\frac{\frac{\mathcal{E}_{\max}}{X_L}}{\frac{\mathcal{E}_{\max}}{R}}\right)=\tan^{-1}\left(\frac{R}{X_L}\right)\qquad(2)$$

Simplify equation (1) and rewrite equation (2) to obtain:

$$I=\sqrt{\left(\frac{\mathcal{E}_{\max}}{R}\right)^2+\left(\frac{\mathcal{E}_{\max}}{X_L}\right)^2}\cos(\omega t-\delta)$$

$$=\mathcal{E}_{\max}\sqrt{\left(\frac{1}{R}\right)^2+\left(\frac{1}{X_L}\right)^2}\cos(\omega t-\delta)$$

$$=\mathcal{E}_{\max}\sqrt{\left(\frac{1}{Z}\right)^2}\cos(\omega t-\delta)$$

$$=\boxed{\frac{\mathcal{E}_{\max}}{Z}\cos(\omega t-\delta)}$$

where

$$\tan\delta=\boxed{\frac{R}{X_L}}\text{ and }\frac{1}{Z^2}=\frac{1}{R^2}+\frac{1}{X_L^2}$$

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Picture the Problem We know that the current leads the voltage across and capacitor and lags the voltage across an inductor. We can use $I_{L,\max}=\mathcal{E}_{\max}/X_L$ and $I_{C,\max}=\mathcal{E}_{\max}/X_C$ to find the amplitudes of these currents. The current in the generator will vanish under resonance conditions, i.e., when $|I_L|=|I_C|$. To find the currents in the inductor and capacitor at resonance, we can use the common potential difference across them and their reactances … together with our knowledge of the phase relationships mentioned above.

(*a*) Express the amplitudes of the currents through the inductor and the capacitor:

$$I_{L,\max}=\frac{\mathcal{E}_{\max}}{X_L}$$

and

$$I_{C,\max}=\frac{\mathcal{E}_{\max}}{X_C}$$

Express X_L and X_C:

$$X_L = \omega L \text{ and } X_C = \frac{1}{\omega C}$$

Substitute to obtain:

$$I_{L,\max} = \frac{100\,\text{V}}{(4\,\text{H})\omega} = \boxed{\frac{25\,\text{V/H}}{\omega}, \text{ lagging } \mathcal{E} \text{ by } 90°}$$

and

$$I_{C,\max} = \frac{100\,\text{V}}{\dfrac{1}{(25\,\mu\text{F})\omega}} = \boxed{\begin{array}{l}(2.5\times10^{-3}\,\text{V}\cdot\text{F})\omega,\\ \text{leading } \mathcal{E} \text{ by } 90°\end{array}}$$

(*b*) Express the condition that $I = 0$:

$$|I_L| = |I_C|$$

or

$$\frac{\mathcal{E}}{\omega L} = \frac{\mathcal{E}}{\dfrac{1}{\omega C}} = \omega C \mathcal{E}$$

Solve for ω to obtain:

$$\omega = \frac{1}{\sqrt{LC}}$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{1}{\sqrt{(4\,\text{H})(25\,\mu\text{F})}} = \boxed{100\,\text{rad/s}}$$

(*c*) Express the current in the inductor at $\omega = \omega_0$:

$$I_L = \left(\frac{25\,\text{V/H}}{100\,\text{s}^{-1}}\right)\cos[(100\,\text{rad/s})t - 90°] = \boxed{(0.250\,\text{A})\sin[(100\,\text{s}^{-1})t]}$$

Express the current in the capacitor at $\omega = \omega_0$:

$$I_C = (2.5\times10^{-3}\,\text{V}\cdot\text{F})(100\,\text{s}^{-1}) \times\cos[(100\,\text{rad/s})t + 90°] = \boxed{-(0.25\,\text{A})\sin[(100\,\text{s}^{-1})t]}$$

***115** ••

Picture the Problem The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find both the average of the voltage squared, $(V^2)_{\text{av}}$ and then use the definition of the rms voltage.

(*a*) From the definition of V_{rms} we have:

$$V_{\text{rms}} = \sqrt{(V_0^2)_{\text{av}}}$$

Noting that $-V_0^2 = V_0^2$, evaluate V_{rms}:

$$V_{\text{rms}} = \sqrt{V_0^2} = V_0 = \boxed{12.0\ \text{V}}$$

(*b*) Noting that the voltage during the second half of each cycle is now zero, express the voltage during the first half cycle of the time interval $\frac{1}{2}\Delta T$:

$$V = V_0$$

Express the square of the voltage during this half cycle:

$$V^2 = V_0^2$$

Calculate $(V^2)_{\text{av}}$ by integrating V^2 from t = 0 to t = $\frac{1}{2}\Delta T$ and dividing by ΔT:

$$(V^2)_{\text{av}} = \frac{V_0^2}{\Delta T}\int_0^{\frac{1}{2}\Delta T} dt = \frac{V_0^2}{\Delta T}[t]_0^{\frac{1}{2}\Delta T} = \tfrac{1}{2}V_0^2$$

Substitute to obtain:

$$V_{\text{rms}} = \sqrt{\tfrac{1}{2}V_0^2} = \frac{V_0}{\sqrt{2}} = \frac{12\ \text{V}}{\sqrt{2}} = \boxed{8.49\ \text{V}}$$

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Picture the Problem We can use the definition of capacitance to find the charge on each capacitor and the definition of current to find the steady-state current in the circuit. We can find the maximum and minimum energy stored in the capacitors using $U = \frac{1}{2}C_{\text{eq}}V^2$, where V is either the maximum or the minimum potential difference across the capacitors.

(*a*) Use the definition of capacitance to express the charge on each capacitor:

$Q_1 = C_1V_1$ and $Q_2 = C_2V_2$

or, because the capacitors are in parallel,

$Q_1 = C_1V$ and $Q_2 = C_2V$

where $V = \mathcal{E} + 24\ \text{V}$

Substitute to obtain:

$$\begin{aligned} Q_1 &= C_1(\mathcal{E} + 24\ \text{V}) \\ &= (3\ \mu\text{F})[(20\ \text{V})\cos(120\pi t) + 24\ \text{V}] \\ &= \boxed{(60\ \mu\text{F})\cos(120\pi t) + 72\ \mu\text{F}} \end{aligned}$$

and

$$Q_2 = C_2(\mathcal{E} + 24\,\text{V})$$
$$= (1.5\,\mu\text{F})[(20\,\text{V})\cos(120\pi t) + 24\,\text{V}]$$
$$= \boxed{(30\,\mu\text{F})\cos(120\pi t) + 36\,\mu\text{F}}$$

(*b*) Express the steady-state current as the rate at which charge is being delivered to the capacitors:

$$I = \frac{dQ}{dt} = \frac{d}{dt}(Q_1 + Q_2)$$

Substitute for Q_1 and Q_2 and evaluate I:

$$I = \frac{d}{dt}[(60\,\mu\text{F})\cos(120\pi t) + 72\,\mu\text{F}$$
$$+ (30\,\mu\text{F})\cos(120\pi t) + 36\,\mu\text{F}]$$
$$= -120\pi(60\,\mu\text{F})\sin(120\pi t)$$
$$-120\pi(30\,\mu\text{F})\sin(120\pi t)$$
$$= \boxed{-(33.9\,\text{mA})\sin(120\pi t)}$$

(*c*) Express U_{max} in terms of the maximum potential difference across the capacitors:

$$U_{max} = \tfrac{1}{2}C_{eq}V_{max}^2$$

Because V_{max} = 44 V and $C_{eq} = C_1 + C_2 = 3\ \mu\text{F} + 1.5\ \mu\text{F} = 4.5\ \mu\text{F}$:

$$U_{max} = \tfrac{1}{2}(4.5\,\mu\text{F})(44\,\text{V})^2 = \boxed{4.36\,\text{mJ}}$$

(*d*) Express U_{min} in terms of the minimum potential difference across the capacitors:

$$U_{min} = \tfrac{1}{2}C_{eq}V_{min}^2$$

The minimum energy stored in the capacitors occurs when $V_{min} = 24\ \text{V} - \mathcal{E}_{max} = 4\ \text{V}$:

$$U_{min} = \tfrac{1}{2}(4.5\,\mu\text{F})(4\,\text{V})^2 = \boxed{36.0\,\mu\text{J}}$$

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Picture the Problem We can apply Kirchhoff's loop rule to express the current in the circuit in terms of the emfs of the sources and the resistance of the resistor. We can then find I_{max} and I_{min} by considering the conditions under which the time-dependent factor in I will be a maximum or a minimum. The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find average of the current squared, $(I^2)_{av}$ and then I_{rms}.

Apply Kirchhoff's loop rule to

$$\mathcal{E}_1 + \mathcal{E}_2 - IR = 0$$

obtain:

Solve for I:

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R}$$

Substitute numerical values to obtain:

$$I = \frac{(20\,\text{V})\cos\left(2\pi\left(180\,\text{s}^{-1}\right)t\right) + 18\,\text{V}}{36\,\Omega}$$

$$= 0.5\,\text{A} + (0.556\,\text{A})\cos\left(1131\,\text{s}^{-1}\right)t$$

Express the condition that must be satisfied if the current is to be a maximum:

$$\cos\left(1131\,\text{s}^{-1}\right)t = 1$$

Evaluate I_{max}:

$$I_{\text{max}} = 0.5\,\text{A} + 0.556\,\text{A} = \boxed{1.06\,\text{A}}$$

Express the condition that must be satisfied if the current is to be a minimum:

$$\cos\left(1131\,\text{s}^{-1}\right)t = -1$$

Evaluate I_{min}:

$$I_{\text{min}} = 0.5\,\text{A} - 0.556\,\text{A} = \boxed{-0.0560\,\text{A}}$$

Because the average value of $\cos\omega t = 0$:

$$I_{\text{av}} = \boxed{0.500\,\text{A}}$$

Express and evaluate the average current delivered by the source whose emf is $\mathcal{E}_2$:

$$I_2 = \frac{\mathcal{E}_2}{R} = \frac{18\,\text{V}}{36\,\Omega} = 0.5\,\text{A}$$

Because $I_1 = (0.556\,\text{A})\cos\left(1131\,\text{s}^{-1}\right)t$:

$$\left(I_1^2\right)_{\text{av}} = \frac{1}{5.56\,\text{ms}}\int_0^{5.56\,\text{ms}} (0.556\,\text{A})^2 \cos^2\left(1131\,\text{s}^{-1}\right)t\,dt$$

Use the trigonometric identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ to obtain:

$$\left(I_1^2\right)_{\text{av}} = \frac{0.309\,\text{A}^2}{2(5.56\,\text{ms})}\int_0^{5.56\,\text{ms}} \left(1 + \cos 2\left(1131\,\text{s}^{-1}\right)t\right)dt$$

$$= \left(27.8\,\text{A}^2/\text{s}\right)\left[t + \frac{1}{2262\,\text{s}^{-1}}\sin\left(2262\,\text{s}^{-1}\right)t\right]_0^{5.56\,\text{ms}}$$

Evaluate $\left(I_1^2\right)_{av}$:

$$\left(I_1^2\right)_{av} = \left(27.8\,\text{A}^2/\text{s}\right)\left[5.56\,\text{ms} + \frac{1}{2262\,\text{s}^{-1}}\sin\left(2262\,\text{s}^{-1}\right)\left(5.56\,\text{ms}\right)\right] = 0.1543\,\text{A}^2$$

Express $\left(I^2\right)_{av}$:

$$\begin{aligned}\left(I^2\right)_{av} &= \left(I_1^2\right)_{av} + \left(I_2^2\right)_{av} \\ &= 0.1543\,\text{A}^2 + \left(0.5\,\text{A}\right)^2 \\ &= 0.4043\,\text{A}^2\end{aligned}$$

Evaluate I_{rms}:

$$\begin{aligned}I_{rms} &= \sqrt{\left(I^2\right)_{av}} = \sqrt{0.4043\,\text{A}^2} \\ &= \boxed{0.636\,\text{A}}\end{aligned}$$

***120** ••

Picture the Problem We can apply Kirchhoff's loop rule to obtain an expression for charge on the capacitor as a function of time. Differentiating this expression with respect to time will give us the current in the circuit. We can then find I_{max} and I_{min} by considering the conditions under which the ti[illegible]ependent factor in I will be a maximum or a minimum. We can use the maximum value of the current to find I_{rms}.

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_1 + \mathcal{E}_2 - \frac{q(t)}{C} = 0$$

Substitute numerical values and solve for $q(t)$:

$$\begin{aligned}q(t) &= \left(2\,\mu\text{F}\right)\left(20\,\text{V}\right)\cos\left(1131\text{s}^{-1}\right)t \\ &\quad + \left(2\,\mu\text{F}\right)\left(18\,\text{V}\right) \\ &= \left(40\,\mu\text{C}\right)\cos\left(1131\text{s}^{-1}\right)t + 36\,\mu\text{C}\end{aligned}$$

Differentiate this expression with respect to t to obtain the current as a function of time:

$$\begin{aligned}I &= \frac{dq}{dt} = \frac{d}{dt}\left[\left(40\,\mu\text{C}\right)\cos\left(1131\text{s}^{-1}\right)t \right. \\ &\quad \left. + 36\,\mu\text{C}\right] \\ &= \boxed{-\left(45.2\,\text{mA}\right)\sin\left(1131\text{s}^{-1}\right)t}\end{aligned}$$

Express the condition that must be satisfied if the current is to be a minimum:

$$\sin\left(1131\text{s}^{-1}\right)t = 1$$

and

$$I_{min} = \boxed{-45.2\,\text{mA}}$$

Express the condition that must be satisfied if the current is to be a

$$\sin\left(1131\text{s}^{-1}\right)t = -1$$

and

maximum:

$$I_{\text{max}} = \boxed{45.2\,\text{mA}}$$

Because the dc source sees the capacitor as an open circuit and the average value of the sine function over a period is zero:

$$I_{\text{av}} = \boxed{0}$$

Because the peak current is 45.2 mA:

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{45.2\,\text{mA}}{\sqrt{2}} = \boxed{32.0\,\text{mA}}$$