HOMEWORK#10 SOLUTION

[Chap28]

***6** •

Determine the Concept The magnetic energy stored in an inductor is given by $U_m = \frac{1}{2}LI^2$. Doubling I quadruples U_m. $\boxed{(c)\text{ is correct.}}$

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Picture the Problem We can use $B = \mu_0 nI$ to find the magnetic field on the axis at the center of the solenoid and the definition of magnetic flux to evaluate ϕ_m. We can use the definition of magnetic flux in terms of L and I to find the self-inductance of the solenoid. Finally, we can use Faraday's law to find the induced emf in the solenoid when the current changes at 150 A/s.

(*a*) Apply the expression for B inside a long solenoid to express and evaluate B:

$$B = \mu_0 nI = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.25\,\text{m}}\right)(3\,\text{A}) = \boxed{6.03\,\text{mT}}$$

(*b*) Apply the definition of magnetic flux to obtain:

$$\phi_m = NBA = (400)(6.03\,\text{mT})\pi(0.01\,\text{m})^2 = \boxed{7.58\times10^{-4}\ \text{Wb}}$$

(*c*) Relate the self-inductance of the solenoid to the magnetic flux through it and its current:

$$L = \frac{\phi_m}{I} = \frac{7.58\times10^{-4}\ \text{Wb}}{3\,\text{A}} = \boxed{0.253\,\text{mH}}$$

(*d*) Apply Faraday's law to obtain:

$$\mathcal{E} = -L\frac{dI}{dt} = -(0.253\,\text{mH})(150\,\text{A/s}) = \boxed{-38.0\,\text{mV}}$$

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Picture the Problem We can find the current using $I = I_f\left(1-e^{-t/\tau}\right)$ where $I_f = \varepsilon_0/R$ and $\tau = L/R$ and its rate of change by differentiating this expression with respect to time.

Express the dependence of the current on I_f and τ:

$$I = I_f\left(1-e^{-t/\tau}\right)$$

Evaluate I_f and τ:

$$I_\mathrm{f} = \frac{\mathcal{E}_0}{R} = \frac{100\ \mathrm{V}}{8\ \Omega} = 12.5\ \mathrm{A}$$

and

$$\tau = \frac{L}{R} = \frac{4\ \mathrm{H}}{8\ \Omega} = 0.5\ \mathrm{s}$$

Substitute to obtain:

$$I = (12.5\ \mathrm{A})\left(1 - e^{-t/0.5\,\mathrm{s}}\right) = (12.5\ \mathrm{A})\left(1 - e^{-2t\,\mathrm{s}^{-1}}\right)$$

Express dI/dt:

$$\frac{dI}{dt} = (12.5\ \mathrm{A})\left(-e^{-2t\,\mathrm{s}^{-1}}\right)\left(-2\ \mathrm{s}^{-1}\right) = (25\ \mathrm{A/s})e^{-2t\,\mathrm{s}^{-1}}$$

(*a*) When $t = 0$:

$$I = (12.5\ \mathrm{A})\left(1 - e^{0}\right) = \boxed{0}$$

and

$$\frac{dI}{dt} = (25\ \mathrm{A/s})e^{0} = \boxed{25.0\ \mathrm{A/s}}$$

(*b*) When $t = 0.1$ s:

$$I = (12.5\ \mathrm{A})\left(1 - e^{-0.2}\right) = \boxed{2.27\ \mathrm{A}}$$

and

$$\frac{dI}{dt} = (25\ \mathrm{A/s})e^{-0.2} = \boxed{20.5\ \mathrm{A/s}}$$

(*c*) When $t = 0.5$ s:

$$I = (12.5\ \mathrm{A})\left(1 - e^{-1}\right) = \boxed{7.90\ \mathrm{A}}$$

and

$$\frac{dI}{dt} = (25\ \mathrm{A/s})e^{-1} = \boxed{9.20\ \mathrm{A/s}}$$

(*d*) When $t = 1.0$ s:

$$I = (12.5\ \mathrm{A})\left(1 - e^{-2}\right) = \boxed{10.8\ \mathrm{A}}$$

and

$$\frac{dI}{dt} = (25\ \mathrm{A/s})e^{-2} = \boxed{3.38\ \mathrm{A/s}}$$

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Picture the Problem If the current is initially zero in an *LR* circuit, its value at some later time *t* is given by $I = I_\mathrm{f}\left(1 - e^{-t/\tau}\right)$, where $I_\mathrm{f} = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the rate of increase of the current by differentiating *I* with respect to time and the time for the current to reach any given fraction of its initial value by solving for *t*.

(*a*) Express the current in the circuit as a function of time:

$$I = \frac{\mathcal{E}_0}{R}\left(1 - e^{-t/\tau}\right)$$

Express the initial rate of increase of the current by differentiating this expression with respect to time:

$$\frac{dI}{dt} = \frac{\mathcal{E}_0}{R}\frac{d}{dt}\left(1 - e^{-t/\tau}\right)$$

$$= \frac{\mathcal{E}_0}{R}\left(-e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) = \frac{\mathcal{E}_0}{\tau R}e^{-\frac{R}{L}t}$$

$$= \frac{\mathcal{E}_0}{L}e^{-\frac{R}{L}t}$$

Evaluate dI/dt at $t = 0$ to obtain:

$$\left.\frac{dI}{dt}\right|_{t=0} = \frac{\mathcal{E}_0}{L}e^0 = \frac{12\,\text{V}}{4\,\text{mH}} = \boxed{3.00\,\text{kA/s}}$$

(*b*) When $I = 0.5I_\text{f}$:

$$0.5 = 1 - e^{-t/\tau} \Rightarrow e^{-t/\tau} = 0.5$$

Evaluate dI/dt with $e^{-t/\tau} = 0.5$ to obtain:

$$\left.\frac{dI}{dt}\right|_{e^{-t/\tau}=0.5} = 0.5\frac{\mathcal{E}_0}{L} = 0.5\left(\frac{12\,\text{V}}{4\,\text{mH}}\right)$$

$$= \boxed{1.50\,\text{kA/s}}$$

(*c*) Calculate I_f from ε and R:

$$I_\text{f} = \frac{\mathcal{E}_0}{R} = \frac{12\,\text{V}}{150\,\Omega} = \boxed{80.0\text{mA}}$$

(*d*) When $I = 0.99I_\text{f}$:

$$0.99 = 1 - e^{-t/\tau} \Rightarrow e^{-t/\tau} = 0.01$$

Solve for and evaluate *t*:

$$t = -\tau\ln(0.01) = -\frac{L}{R}\ln(0.01)$$

$$= -\frac{4\,\text{mH}}{150\,\Omega}\ln(0.01) = \boxed{0.123\,\text{ms}}$$

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Picture the Problem The self-induced emf in the inductor is proportional to the rate at which the current through it is changing. Under steady-state conditions, $dI/dt = 0$ and so the self-induced emf in the inductor is zero. We can use Kirchhoff's loop rule to obtain the current through and the voltage across the inductor as a function of time.

(*a*) Because, under steady-state conditions, the self-induced emf in

$$10\,\text{V} - (10\,\Omega)I = 0$$

and

the inductor is zero and because the inductor has negligible resistance, we can apply Kirchhoff's loop rule to the loop that includes the source, the 10-Ω resistor, and the inductor to find the current drawn from the battery and flowing through the inductor and the 10-Ω resistor:

$$I = \frac{10\,\text{V}}{10\,\Omega} = \boxed{1.00\,\text{A}}$$

By applying Kirchhoff's junction rule at the junction between the resistors, we can conclude that:

$$I_{100\text{-}\Omega\,\text{resistor}} = I_{\text{battery}} - I_{\text{inductor}} = \boxed{0}$$

(*b*) When the switch is closed, the current cannot immediately go to zero in the circuit because of the inductor. For a time, a current will circulate in the circuit loop between the inductor and the 100-Ω resistor. Because the current flowing through this circuit is initially 1 A, the voltage drop across the 100-Ω resistor is initially $\boxed{100\,\text{V.}}$ Conservation of energy (Kirchhoff's loop rule) requires that the voltage drop across the inductor is also $\boxed{100\,\text{V.}}$

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Picture the Problem The current in an initially energized but source-free *RL* circuit is given by $I = I_0 e^{-t/\tau}$. We can find τ from this equation and then use its definition to evaluate *L*.

(*a*) Express the current in the *RL* circuit as a function of time:

$$I = I_0 e^{-t/\tau}$$

Solve for and evaluate τ:

$$\tau = -\frac{t}{\ln\left(\frac{I}{I_0}\right)} = -\frac{45\,\text{ms}}{\ln\left(\frac{1.5\,\text{A}}{2.5\,\text{A}}\right)} = \boxed{88.1\,\text{ms}}$$

(*b*) Using the definition of the inductive time constant, relate *L* to *R*:

$$L = \tau R$$

Substitute numerical values and evaluate *L*:

$$L = (0.0881\,\text{s})(0.4\,\Omega) = \boxed{35.2\,\text{mH}}$$

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Picture the Problem We can integrate $dE/dt = \mathcal{E}_0 I,$ where $I = I_\mathrm{f}\left(1-e^{-t/\tau}\right)$, to find the energy supplied by the battery, $dE_\mathrm{J}/dt = I^2R$ to find the energy dissipated in the resistor, and $U_L(\tau) = \frac{1}{2}L(I(\tau))^2$ to express the energy that has been stored in the inductor when $t = \tau$.

(*a*) Express the rate at which energy is supplied by the battery:

$$\frac{dE}{dt} = \mathcal{E}_0 I$$

Express the current in the circuit as a function of time:

$$I = \frac{\mathcal{E}_0}{R}\left(1-e^{-t/\tau}\right)$$

Substitute to obtain:

$$\frac{dE}{dt} = \frac{\mathcal{E}_0^2}{R}\left(1-e^{-t/\tau}\right)$$

Separate variables and integrate from $t = 0$ to $t = \tau$ to obtain:

$$E = \frac{\mathcal{E}_0^2}{R}\int_0^\tau\left(1-e^{-t/\tau}\right)dt$$

$$= \frac{\mathcal{E}_0^2}{R}\left[\tau-\left(-\tau e^{-1}+\tau\right)\right]$$

$$= \frac{\mathcal{E}_0^2}{R}\frac{\tau}{e} = \frac{\mathcal{E}_0^2 L}{R^2 e}$$

Substitute numerical values and evaluate E:

$$E = \frac{(12\,\mathrm{V})^2(0.6\,\mathrm{H})}{(3\,\Omega)^2 e} = \boxed{3.53\,\mathrm{J}}$$

(*b*) Express the rate at which energy is being dissipated in the resistor:

$$\frac{dE_\mathrm{J}}{dt} = I^2R = \left[\frac{\mathcal{E}_0}{R}\left(1-e^{-t/\tau}\right)\right]^2 R$$

$$= \frac{\mathcal{E}_0^2}{R}\left(1-2e^{-t/\tau}+e^{-2t/\tau}\right)$$

Separate variables and integrate from $t = 0$ to $t = \tau$ to obtain:

$$E_\mathrm{J} = \frac{\mathcal{E}_0^2}{R}\int_0^\tau\left(1-2e^{-t/\tau}+e^{-2t/\tau}\right)dt$$

$$= \frac{\mathcal{E}_0^2}{R}\left(\frac{2\tau}{e}-\frac{\tau}{2}-\frac{\tau}{2e^2}\right)$$

$$= \frac{\mathcal{E}_0^2 L}{R^2}\left(\frac{2}{e}-\frac{1}{2}-\frac{1}{2e^2}\right)$$

Substitute numerical values and evaluate E_J:

$$E_J = \frac{(12\,\text{V})^2(0.6\,\text{H})}{(3\,\Omega)^2}\left(\frac{2}{e}-\frac{1}{2}-\frac{1}{2e^2}\right) = \boxed{1.61\,\text{J}}$$

(*c*) Express the energy stored in the inductor when $t = \tau$:

$$U_L(\tau) = \tfrac{1}{2}L(I(\tau))^2 = \tfrac{1}{2}L\left(\frac{\mathcal{E}_0}{R}\left(1-e^{-1}\right)\right)^2 = \frac{L\mathcal{E}_0^2}{2R^2}\left(1-e^{-1}\right)^2$$

Substitute numerical values and evaluate E_L:

$$U_L(\tau) = \frac{(0.6\,\text{H})(12\,\text{V})^2}{2(3\,\Omega)^2}\left(1-e^{-1}\right)^2 = \boxed{1.92\,\text{J}}$$

Remarks: Note that, as we would expect from energy conservation, $E = E_J + E_L$.

[Honors section]

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(*c*) Apply Kirchhoff's loop rule to the *RL* circuit to obtain:

$$L\frac{dI}{dt}+IR=0$$

The solution to this differential equation is:

$$I(t)=I_0e^{-\frac{R}{L}t}=I_0e^{-\frac{t}{\tau}}$$

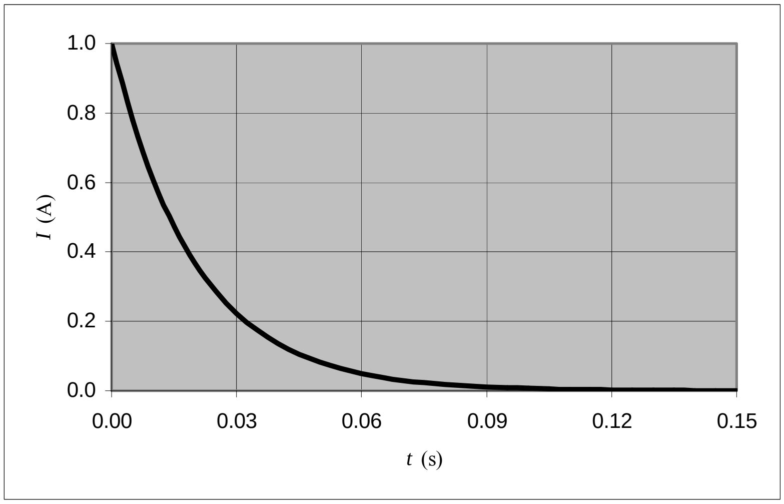
$$\text{where }\tau=\frac{L}{R}=\frac{2\,\text{H}}{100\,\Omega}=0.02\,\text{s}$$

A spreadsheet program to generate the data for graphs of the current and the voltage across the inductor as functions of time is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	2	L
B2	100	R
B3	1	I_0
A6	0	t_0
B6	$B$3*EXP((–$B$2/$B$1)*A6)	$I_0e^{-\frac{R}{L}t}$

	A	B	C
1	L=	2	H
2	R=	100	ohms
3	I_0=	1	A
4			
5	t	I(t)	V(t)
6	0.000	1.00E+00	100.00
7	0.005	7.79E–01	77.88
8	0.010	6.07E–01	60.65
9	0.015	4.72E–01	47.24
10	0.020	3.68E–01	36.79
11	0.025	2.87E–01	28.65
12	0.030	2.23E–01	22.31
32	0.130	1.50E–03	0.15
33	0.135	1.17E–03	0.12
34	0.140	9.12E–04	0.09
35	0.145	7.10E–04	0.07
36	0.150	5.53E–04	0.06

The following graph of the current in the inductor as a function of time was plotted using the data in columns A and B of the spreadsheet program.



The following graph of the voltage across the inductor as a function of time was plotted using the data in columns A and C of the spreadsheet program.