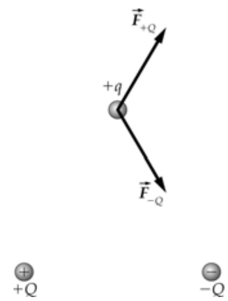
HOMEWORK#1 SOLUTION

3 ••

Determine the Concept During this sequence of events, negative charges are attracted from ground to the rectangular metal plate B. When S is opened, these charges are trapped on B and remain there when the charged body is removed. Hence B is negatively charged and $\boxed{(c)\ \text{is correct.}}$

6 •

Determine the Concept The forces acting on +q are shown in the diagram. The force acting on +q due to –Q is along the line joining them and directed toward –Q. The force acting on +q due to +Q is along the line joining them and directed away from +Q.



Because charges +Q and –Q are equal in magnitude, the forces due to these charges are equal and their sum (the net force on +q) will be to the right and so $\boxed{(e)\ \text{is correct.}}$ Note that the vertical components of these forces add up to zero.

***8** •

Determine the Concept $\vec{E}$ is zero wherever the net force acting on a test charge is zero. At the center of the square the two positive charges alone would produce a net electric field of zero, and the two negative charges alone would also produce a net electric field of zero. Thus, the net force acting on a test charge at the midpoint of the square will be zero. $\boxed{(b)\ \text{is correct.}}$

35 ••

Picture the Problem By considering the symmetry of the array of charges we can see that the y component of the force on q is zero. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on q.

Express the net force acting on q:

$$\vec{F}_q = \vec{F}_{Q\,\text{on}\,x\,\text{axis},q} + 2\vec{F}_{Q\,\text{at}\,45°,q}$$

Express the force on q due to the charge Q on the x axis:

$$\vec{\mathbf{F}}_{Q\,\text{on}\,x\,\text{axis},q} = \frac{kqQ}{R^2}\hat{\mathbf{i}}$$

Express the net force on q due to the charges at 45°:

$$2\vec{\mathbf{F}}_{Q\,\text{at}\,45°,q} = 2\frac{kqQ}{R^2}\cos 45°\,\hat{\mathbf{i}} = \frac{2}{\sqrt{2}}\frac{kqQ}{R^2}\hat{\mathbf{i}}$$

Substitute to obtain:

$$\vec{\mathbf{F}}_q = \frac{kqQ}{R^2}\hat{\mathbf{i}} + \frac{2}{\sqrt{2}}\frac{kqQ}{R^2}\hat{\mathbf{i}} = \frac{kqQ}{R^2}(1+\sqrt{2})\hat{\mathbf{i}}$$

40 •

Picture the Problem We can compare the electric and gravitational forces acting on an electron by expressing their ratio. We can equate these forces to find the charge that would have to be placed on a penny in order to balance the earth's gravitational force on it.

(*a*) Express the magnitude of the electric force acting on the electron:

$$F_e = eE$$

Express the magnitude of the gravitational force acting on the electron:

$$F_g = m_e g$$

Express the ratio of these forces to obtain:

$$\frac{F_e}{F_g} = \frac{eE}{mg}$$

Substitute numerical values and evaluate F_e/F_g:

$$\frac{F_e}{F_g} = \frac{(1.6\times10^{-19}\,\text{C})(150\,\text{N/C})}{(9.11\times10^{-31}\,\text{kg})(9.81\,\text{m/s}^2)} = 2.69\times10^{12}$$

or

$\boxed{F_e = (2.69\times10^{12})F_g}$, i.e., the electric force is greater by a factor of 2.69×10^{12}.

(*b*) Equate the electric and gravitational forces acting on the penny and solve for q to obtain:

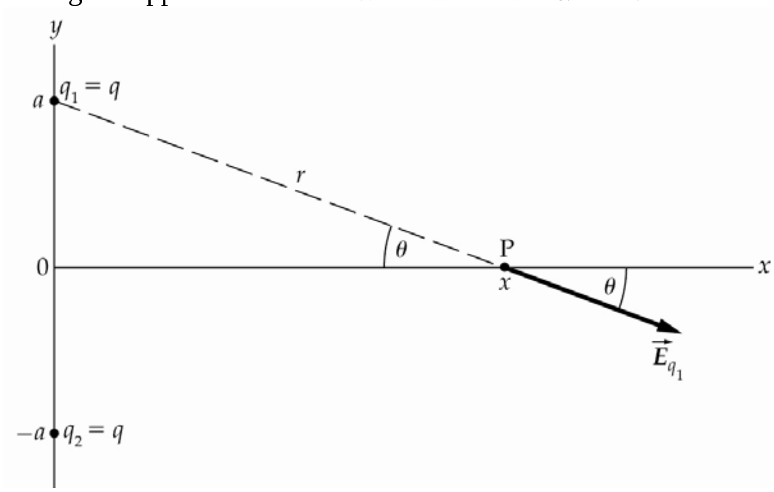
$$q = \frac{mg}{E}$$

Substitute numerical values and evaluate q:

$$q = \frac{(3\times10^{-3}\,\text{kg})(9.81\,\text{m/s}^2)}{150\,\text{N/C}} = \boxed{1.96\times10^{-4}\,\text{C}}$$

44 ••

Picture the Problem The diagram shows the locations of the charges q_1 and q_2 and the point on the *x* axis at which we are to find $\vec{E}$. From symmetry considerations we can conclude that the *y* component of $\vec{E}$ at any point on the *x* axis is zero. We can use Coulomb's law for the electric field due to point charges to find the field at any point on the *x* axis. We can establish the results called for in parts (*b*) and (*c*) by factoring the radicand and using the approximation $1+\alpha \approx 1$ whenever $\alpha << 1$.



(*a*) Express the *x*-component of the electric field due to the charges at *y* = *a* and *y* = –*a* as a function of the distance *r* from either charge to point P:

$$\vec{E}_x = 2\frac{kq}{r^2}\cos\theta\,\hat{\boldsymbol{i}}$$

Substitute for $\cos\theta$ and *r* to obtain:

$$\vec{E}_x = 2\frac{kq}{r^2}\frac{x}{r}\hat{\boldsymbol{i}} = \frac{2kqx}{r^3}\hat{\boldsymbol{i}} = \frac{2kqx}{\left(x^2+a^2\right)^{3/2}}\hat{\boldsymbol{i}}$$

$$= \frac{2kqx}{\left(x^2+a^2\right)^{3/2}}\hat{\boldsymbol{i}}$$

and

$$E_x = \boxed{\frac{2kqx}{\left(x^2+a^2\right)^{3/2}}}$$

(*b*) For $|x| << a, x^2 + a^2 \approx a^2$, so:

$$E_x \approx \frac{2kqx}{\left(a^2\right)^{3/2}} = \boxed{\frac{2kqx}{a^3}}$$

For $|x| >> a, x^2 + a^2 \approx x^2$, so:

$$E_x \approx \frac{2kqx}{\left(x^2\right)^{3/2}} = \boxed{\frac{2kq}{x^2}}$$

(c) For $x >> a$, the charges separated by a would appear to be a single charge of magnitude $2q$. Its field would be given by $E_x = \dfrac{2kq}{x^2}$.

Factor the radicand to obtain:

$$E_x = 2kqx\left[x^2\left(1+\frac{a^2}{x^2}\right)\right]^{-3/2}$$

For $a << x$:

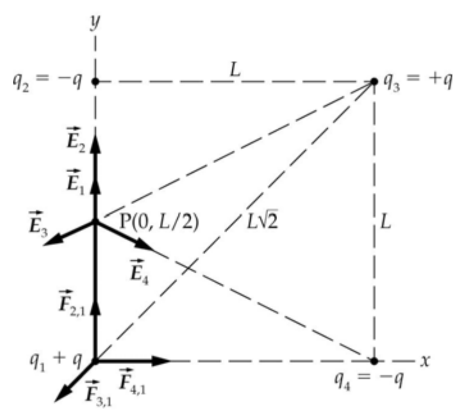
$$1+\frac{a^2}{x^2} \approx 1$$

and

$$E_x = 2kqx\left[x^2\right]^{-3/2} = \boxed{\frac{2kq}{x^2}}$$

80 ••

Picture the Problem Let the origin be at the lower left-hand corner and designate the charges as shown in the diagram. We can apply Coulomb's law for point charges to find the forces exerted on q_1 by q_2, q_3, and q_4 and superimpose these forces to find the net force exerted on q_1. In part (*b*), we'll use Coulomb's law for the electric field due to a point charge and the superposition of fields to find the electric field at point $P(0, L/2)$.



(*a*) Using the superposition of forces, express the net force exerted on q_1:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1}$$

Apply Coulomb's law to express $\vec{F}_{2,1}$:

$$\vec{F}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2}\hat{r}_{2,1} = \frac{kq_2q_1}{r_{2,1}^3}\vec{r}_{2,1} = \frac{k(-q)q}{L^3}\left(-L\,\hat{j}\right) = \frac{kq^2}{L^2}\hat{j}$$

Apply Coulomb's law to express $\vec{F}_{4,1}$:

$$\vec{F}_{4,1} = \frac{kq_4q_1}{r_{4,1}^2}\hat{r}_{4,1} = \frac{kq_4q_1}{r_{4,1}^3}\vec{r}_{4,1} = \frac{k(-q)q}{L^3}\left(-L\,\hat{i}\right) = \frac{kq^2}{L^2}\hat{i}$$

Apply Coulomb's law to express $\vec{F}_{3,1}$:

$$\vec{F}_{3,1} = \frac{kq_3q_1}{r_{3,1}^2}\hat{r}_{3,1} = \frac{kq_3q_1}{r_{3,1}^3}\vec{r}_{3,1}$$
$$= \frac{kq^2}{2^{3/2}L^3}\left(-L\,\hat{i} - L\,\hat{j}\right)$$
$$= -\frac{kq^2}{2^{3/2}L^2}\left(\hat{i} + \hat{j}\right)$$

Substitute and simplify to obtain:

$$\vec{F}_1 = \frac{kq^2}{L^2}\hat{j} - \frac{kq^2}{2^{3/2}L^2}\left(\hat{i} + \hat{j}\right) + \frac{kq^2}{L^2}\hat{i}$$
$$= \frac{kq^2}{L^2}\left(\hat{i} + \hat{j}\right) - \frac{kq^2}{2^{3/2}L^2}\left(\hat{i} + \hat{j}\right)$$
$$= \boxed{\frac{kq^2}{L^2}\left(1 - \frac{1}{2\sqrt{2}}\right)\left(\hat{i} + \hat{j}\right)}$$

(*b*) Using superposition of fields, express the resultant field at point *P*:

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \qquad (1)$$

Use Coulomb's law to express $\vec{E}_1$:

$$\vec{E}_1 = \frac{kq_1}{r_{1,P}^2}\hat{r}_{1,P} = \frac{kq}{r_{1,P}^3}\left(\frac{L}{2}\hat{j}\right)$$
$$= \frac{kq}{\left(\frac{L}{2}\right)^3}\left(\frac{L}{2}\hat{j}\right) = \frac{4kq}{L^2}\hat{j}$$

Use Coulomb's law to express $\vec{E}_2$:

$$\vec{E}_2 = \frac{kq_2}{r_{2,P}^2}\hat{r}_{2,P} = \frac{k(-q)}{r_{2,P}^3}\left(\frac{L}{2}\hat{j}\right)$$
$$= \frac{-kq}{\left(\frac{L}{2}\right)^3}\left(-\frac{L}{2}\hat{j}\right) = \frac{4kq}{L^2}\hat{j}$$

Use Coulomb's law to express $\vec{E}_3$:

$$\vec{E}_3 = \frac{kq_3}{r_{3,P}^2}\hat{r}_{3,P} = \frac{kq}{r_{3,P}^3}\left(-L\hat{i} - \frac{L}{2}\hat{j}\right)$$
$$= \frac{8kq}{5^{3/2}L^2}\left(-\hat{i} - \frac{1}{2}\hat{j}\right)$$

Use Coulomb's law to express $\vec{E}_4$:

$$\vec{\boldsymbol{E}}_4 = \frac{kq_4}{r_{4,P}^2}\hat{\boldsymbol{r}}_{3,P} = \frac{k(-q)}{r_{4,P}^3}\left(L\hat{\boldsymbol{i}} - \frac{L}{2}\hat{\boldsymbol{j}}\right)$$

$$= \frac{8kq}{5^{3/2}L^2}\left(\hat{\boldsymbol{i}} - \frac{1}{2}\hat{\boldsymbol{j}}\right)$$

Substitute in equation (1) and simplify to obtain:

$$\vec{\mathbf{E}}_P = \frac{4kq}{L^2}\hat{\mathbf{j}} + \frac{4kq}{L^2}\hat{\mathbf{j}} + \frac{8kq}{5^{3/2}L^2}\left(-\hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}}\right) + \frac{8kq}{5^{3/2}L^2}\left(\hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}}\right) = \boxed{\frac{8kq}{L^2}\left(1 - \frac{\sqrt{5}}{25}\right)\hat{\mathbf{j}}}$$